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ANDISORD: A COMPUTER PROGRAM FOR ATMOSPHERIC AND ENGINEERING RADIATIVE TRANSFER APPLICATIONS

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SUMMARY

In this paper, a radiation transfer algorithm for the solution of the equation of radiative transfer based on a finite difference implementation of the discrete-ordinates method is presented. Given the finite difference formulation of the resulting computer program, (ANDISORD), extension of the current 1-D version to 2-D or 3-D situations is straight forward. The program can handle a wide variety of boundary conditions, it has built-in anisotropic capability, it allows for thermal sources, and it can interact effortlessly with other modes of heat transfer. These characteristics make ANDISORD an excellent tool in engineering applications. Results obtained from benchmarking tests in both atmospheric and engineering applications are given.

INTRODUCTION

Rainfall over the tropics and oceans is one of the most important variables that needs to be estimated in order to properly calibrate the performance of the global circulation models. Given the spatial and temporal variability of rainfall, satellites represent the only viable alternative for obtaining such estimates. At the core of any satellite rainfall retrieval scheme remains the problem of solving, efficiently, the inverse radiation problem which in turn relies on a radiative transfer algorithm.

It is of interest to develop a general model for the solution of the radiative transfer equation in multidimensional, anisotropic, inhomogeneous medium [1]. As a first step in this direction, a radiation transfer algorithm for the solution of the equation of radiative transfer based on a finite difference implementation of the discrete-ordinates method is presented. Given the finite difference formulation of the resulting computer program, (ANDISORD), extension of the current 1D version to 2D or 3D situations is straight forward. The model contains all the procedures necessary to evaluate the optical properties of polydispersions. These polydispersions can be directly related to rainfall through a Marshall-Palmer type of relationship.

The model can handle a wide variety of boundary conditions, it has built-in anisotropic capability, it allows for thermal sources, it has the capability to evaluate the optical properties of particulated media, and it can interact effortlessly with general application programs for the solution of the conservative equations of momentum, mass and energy. These characteristics make ANDISORD an excellent tool in engineering applications.

Results obtained from benchmarking tests in both atmospheric and engineering applications are given in the following sections. Comparisons with other algorithms show the great flexibility and accuracy of ANDISORD.

THE GENERAL EQUATIONS

The ultimate objective of the described work is to develop a radiative transfer model that could be applied to cloudy and rainy atmospheres. This implies that the method used to solve the problem should account for: 1) multiple dimensions (3D); 2) anisotropic scattering (including water droplets); 3) compatibility with other modes of heat transfer; 4) inhomogeneities; 5) real gas; 6) computational efficiency; and 7) accuracy and stability.

The first five conditions are desirable due to the geometry, composition, and radiative properties of clouds themselves. For the sixth condition, computational efficiency, high speed is required because it is foreseen that the method will be applied to the interpretation of satellite data. As for the last condition, accuracy and stability, it is understood that the extra effort necessary to achieve "exact" solutions, although desirable in some situations, is not justified in this type of atmospheric problem due to the inexact knowledge of cloud constituents or their vertical distribution [2]. Hence, good, stable approximations rather than "exact" solutions are sought.

Subject to the boundary conditions of the particular problem at hand, the system of equations governing the problem are:

Energy Conservation Within the Medium. The well known form of the general energy conservation equation follows [3]:

$$\rho C_p \frac{DT}{Dt} = \beta_c T \frac{DP}{Dt} + \nabla \cdot (k\, \nabla T - q_r) + \dot{q} + \Phi' \qquad (1)$$

The left hand side of equation (1) represents the convection and transient energy storage while the terms on the right hand side are, in order, compression work, conduction, net radiative energy gained per unit volume, local heat generation, and viscous dissipation.

The divergence of the heat flux vector. In terms of radiative intensities, the divergence of the heat flux vector is expressed as:

$$\nabla \cdot q_r = \int_0^{\infty} \int_0^{4\pi} \left(\frac{\partial I_\lambda}{\partial x} \mu + \frac{\partial I_\lambda}{\partial y} \delta + \frac{\partial I_\lambda}{\partial z} \gamma \right) d\omega\, d\lambda \qquad (2)$$

where ω denotes solid angle; and μ, δ, and γ are the cosines of the angle between the direction of I (the intensity) and the x, y, and z axis, respectively, and I_λ is the monochromatic intensity at wavelength λ.

The monochromatic intensity. The monochromatic intensity is expressed as:

$$\frac{dI_\lambda}{d\zeta} = -(a_\lambda + s_\lambda) I_\lambda(\zeta) + a_\lambda I_{b\lambda}(\zeta) + \frac{s_\lambda}{4\pi} \int_0^{4\pi} I_\lambda(\zeta, \omega_i)\, \Phi(\lambda, \omega, \omega_i)\, d\omega_i \qquad (3)$$

where I is the radiant intensity, β is the extinction coefficient, I_b is the blackbody intensity, ζ is the line of sight of incident radiation, ω and ω_i are, respectively, the outgoing and the incident solid angle, and Φ is the scattering phase function. The left hand side of equation (3) is the variation of the monochromatic intensity along the line of sight direction (LSD). The three terms on the right hand side are, respectively, the attenuation of monochromatic intensity along the LSD due to the absorption and outward scattering characteristics of the medium, augmentation of monochromatic intensity into the LSD due to emission in the medium, and the augmentation of monochromatic intensity due to inward scattering of the incoming radiant energy into the LSD. The monochromatic absorption and scattering coefficients are a_λ and s_λ respectively.

SOLUTION OF THE GENERAL EQUATIONS

Selection of a Method. A comparison among the different methods to solve the radiative transfer equation (RTE) is tabulated in Byun [4]. He classifies the various methods into four categories: flux (subdivided into flux, P-N or differential approximation, and S-N or discrete ordinates), integral, Monte Carlo, and zone methods. For three dimensional, emitting, absorbing and anisotropically scattering media the P-N and the S-N methods may be selected as those that better satisfy the needs expressed above.

Although the P-N method (in particular P-3) has been formulated and applied to 3D radiation transfer problems with anisotropic scattering conditions [5,6], the results seem to be very poor and questionable [7]. On the other hand, the use of the P-3 method for 2D geometries is claimed to be already too cumbersome. This leaves the S-N method as the best choice for the problem.

In situations where scattering is important, which is the case in the presence of raindrops and cloud water, the discrete ordinates method (S-N) is reported to work very well [8]. Very good agreement is reported [7, 9] between the S-N method (for N equal to or greater than 4) and the zone method for 3D problems with isotropic scattering.

The discrete ordinates method [7, 9, 10] is, therefore, selected as the working tool. The pros and cons for the use of the discrete ordinates method in atmospheric applications are discussed in detail in Lenoble [11] and Siegfrid [12].

The Discrete Ordinates Method. The general formulation as well as the description of the numerical implementation of the discrete ordinates method in three dimensional, anisotropically scattering radiative heat transfer problems is readily available elsewhere [7, 9, 10]. Some modifications have, however, been introduced here in order to make the scheme more applicable to atmospheric problems. In particular: parallel beam radiation (solar beam) and isotropic radiation on the boundaries (background radiation) have been included.

For the control volume depicted in Figure 1 the face intensities are related to the volume-center intensity throughout a spatial interpolation of the form:

$$I_i^p = \alpha\, I_i^{xe} + (1-\alpha)\, I_i^{xr} = \alpha\, I_i^{ye} + (1-\alpha)\, I_i^{yr} = \alpha\, I_i^{ze} + (1-\alpha)\, I_i^{zr} \qquad (4)$$

where α is a weight factor ($0.5 \le \alpha \le 1$) and the superscripts "r" and "e" denote reference and end-face (to indicate where the energy originates and where it arrives) for the indicated coordinate direction.

Equation (3) is discretized and rewritten as:

$$I_i^p = \frac{|\mu_i| A_{n,s} I_i^{xr} + |\delta_i| A_{e,w} I_i^{yr} + |\gamma_i| A_{f,b} I_i^{zr} + \alpha\,(S_1+S_2+S_3)\Delta V_p}{|\mu_i| A_{n,s} + |\delta_i| A_{e,w} + |\gamma_i| A_{f,b} + \alpha\beta\Delta V_p} \qquad (5)$$

where μ_i, δ_i, and γ_i are the direction cosines for the discrete direction i, and where the face areas (A), and the source terms (S_1, S_2, and S_3) for a differential volume ($\Delta V_p = \Delta x\ \Delta y\ \Delta z$) are given by the expressions:

$A_{n,s} = \Delta y\ \Delta z$ north/south faces

$A_{e,w} = \Delta x\ \Delta z$ east/west faces

$A_{f,b} = \Delta y\ \Delta x$ front/back faces

$$S_1 = (1-\Omega)\,\beta\, I_b^p \;\Rightarrow\; S_1 = a I_b^p \qquad (6)$$

$$S_2 = \frac{s}{4\pi} \sum_j w_j\, I_j^p\, \Phi_{ij} \qquad (7)$$

$$S_3 = \frac{s}{4\pi}\, F_o^p\, \Phi_{io} \qquad (8)$$

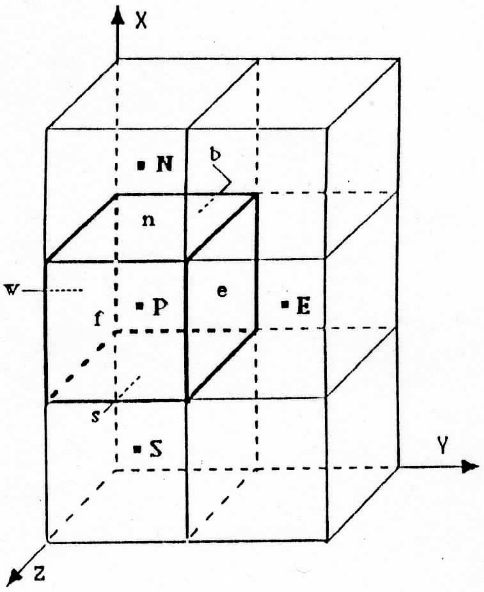

Fig. 1. Three-dimensional Control Volume.

In Equations (6-8) Ω is the scattering albedo; w are the weights for the quadrature procedure; F_o is the external flux for a parallel beam in direction μ_o, δ_o, and γ_o; and Φ_{ij} is the phase function.

The phase function is represented by:

$$\Phi_{ij} = \sum_{n=0}^{N} (2n+1)\, b_n\, P_n(\mu_i\mu_j, \delta_i\delta_j, \gamma_i\gamma_j) \qquad (9)$$

where b_n are coefficients for the series expansion of the phase function in terms of Legendre polynomials of order n (P_n).

When the surface bounding the enclosure is gray, opaque, and emits and reflects diffusively, the boundary conditions for equation (5) are

$$\begin{aligned}
&\text{at}\quad x = 0 && I_i = \varepsilon I_b + \frac{(1-\varepsilon)}{\pi} \sum_{\mu_j<0} w_j\, |\mu_j|\, I_j && \text{for } \mu_i>0\\
&\text{at}\quad x = L_x && I_i = \varepsilon I_b + \frac{(1-\varepsilon)}{\pi} \sum_{\mu_j>0} w_j\, \mu_j\, I_j && \text{for } \mu_i<0\\
&\text{at}\quad y = 0 && I_i = \varepsilon I_b + \frac{(1-\varepsilon)}{\pi} \sum_{\delta_j<0} w_j\, |\delta_j|\, I_j && \text{for } \delta_i>0\\
&\text{at}\quad y = L_y && I_i = \varepsilon I_b + \frac{(1-\varepsilon)}{\pi} \sum_{\delta_j>0} w_j\, \delta_j\, I_j && \text{for } \delta_i<0\\
&\text{at}\quad z = 0 && I_i = \varepsilon I_b + \frac{(1-\varepsilon)}{\pi} \sum_{\gamma_j<0} w_j\, |\gamma_j|\, I_j && \text{for } \gamma_i>0\\
&\text{at}\quad z = L_z && I_i = \varepsilon I_b + \frac{(1-\varepsilon)}{\pi} \sum_{\gamma_j>0} w_j\, \gamma_j\, I_j && \text{for } \gamma_i<0
\end{aligned} \qquad (10)$$

where ε is the surface emissivity and L_i is the total length of the domain in the direction i.

If an external intensity I_s is incident on any of the surfaces, the term I_s should be added to the boundary condition at that surface.

Solution Procedure. If the temperature distribution throughout the computational domain is known, the terms S_l as well as the optical properties of the medium can be evaluated as needed and the system of equations (4-10) can be solved independently of the energy equation. Normally, this is not the case and the radiative transfer equation has to be solved iteratively with the energy equation (*global iterations*) in order to find the needed temperatures.

For steady state, the energy equation (equation (1)) for a moving continuum can be written in the following general form:

$$C(T) = D(T) + S(T) \tag{11}$$

where C(T) is the convective term, D(T) the diffusion term, and S(T) the source term.

The source term can include compression work, viscous dissipation, heat generation and a radiation source. Whatever the other constituents of the source might be, the radiant source, if present, has to be found.

The radiant source is evaluated at each control volume (p) as:

$$(\nabla . q_r)_p = a\left(4\pi I_b^p - \sum_{j=1}^{n} w_j I_j\right) \tag{12}$$

Equation (11) is normally non-linear and must be solved iteratively by a numerical procedure [13]. Applications involving complete *global iterations* are not commonly found in the literature. In most cases, conduction and convection are assumed negligible and the source is considered to be either a radiation source alone ($\nabla . q_r = 0$) or radiation plus generation sources ($\nabla . q_r = q$).

In order to solve for a general energy equation, an iterative procedure between equations (4-10) and equations (11-12) has to be applied.

It is out of the scope of this paper to describe the details or to give examples of problems involving global iterations. However, given the importance of this type of problems in engineering, the interested reader is referred to Sánchez. et al. [14] were an example of the procedure for global iterations in a two-dimensional geometry is given.

THE ONE DIMENSIONAL IMPLEMENTATION

As a first step toward the full solution of the problem, a one-dimensional, parallel layers version of the procedure described previously was implemented. The geometry solved by the program is shown in Figure 2.

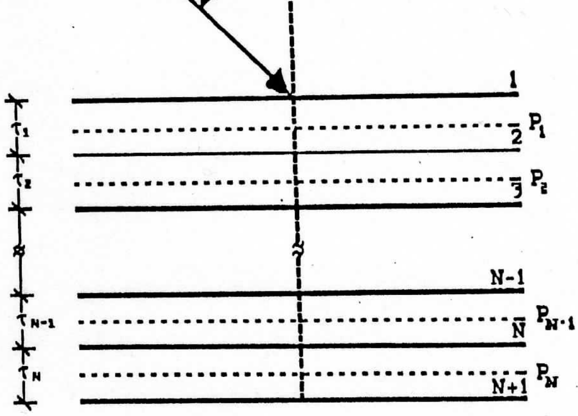

Fig. 2. Parallel Plates Geometry

The main characteristics of the program (called ANDISORD, and written in FORTRAN 77) are:

Sources. The program allows for any combination of parallel beam radiation at any angle θ_0, isotropic background radiation, and thermal sources.

Homogeneity. The layers are considered homogeneous in the horizontal direction, but they can be inhomogeneous in the vertical direction. i.e., the optical thickness (τ), the scattering albedo (ω_o) and the phase function can vary from layer to layer.

Scattering. The program allows for isotropic, anisotropic by similarity, Mie, or Henyey-Greenstein scattering.

Quadrature scheme. The code uses a Gaussian quadrature scheme to solve the integrals.

Azimuthal distribution. The program solves for azimuthally independent radiation. No effort was made in order to implement the azimuthal distribution of radiances, since this would require a Fourier expansion of the intensities. In the future, when the three-dimensional quadrature points are implemented, the azimuthal distribution will be obtained without extra effort due to the spatial distribution of the direction cosines.

Output. The program evaluates the radiances at any level and at any polar angle; the heat fluxes in and out of each inter-layer plane, discriminating between direct and diffuse fluxes; the irradiation at each inter-layer; and the transmissivity, reflectivity, and absorptivity of the whole "atmosphere".

The optical properties. Of primary importance for the application of ANDISORD to the problem of rainfall retrieval is the determination of the optical properties of polydispersions. To this effect, a companion program was written (POLY) that allows for the evaluation of those optical properties based on the following formulas [15, 16]:

$$\beta = \int_{r_{min}}^{r_{max}} \sigma_{ext}(r)\,\frac{dn(r)}{dr}\,dr \tag{13}$$

$$s = \int_{r_{min}}^{r_{max}} \sigma_{sca}(r)\,\frac{dn(r)}{dr}\,dr \tag{14}$$

$$\Phi(\theta) = \frac{1}{s}\int_{r_{min}}^{r_{max}} \sigma_{sca}(r)\,\Phi(\theta,r)\,\frac{dn(r)}{dr}\,dr \tag{15}$$

$$\frac{dn(r)}{dr} = 16000e^{-\Lambda r}\ [m^{-3}\,mm^{-1}] \tag{16}$$

where σ_{ext}, and σ_{sca} are, respectively, the extinction and scattering cross-sections for a single sphere; β is the extinction coefficient (a + s) for the polydispersion; r is the drop radius; n(r) is the drop size distribution; θ is the scattering angle and Λ is:

$$\Lambda = 8.2\,R^{-0.21}\ [mm^{-1}] \tag{17}$$

R being the rainfall rate in $[mm\ hr^{-1}]$. Equations (16-17) imply a classic Marshall and Palmer type drop distribution.

The extinction (σ_{ext}) and scattering (σ_{sca}) cross-sections are evaluated from the exact Mie calculations, for a single hydrometeor of radius r, using wavelength dependent index of refractions tabulated from Kondratyev [17] and the computational procedure of Dave [18]. Expansion of the phase function in a series of Legendre polynomials – to find the b_n terms in equation (9) – is performed by means of a procedure similar to that described by Kumar [19].

RESULTS AND DISCUSSION

In order to benchmark the program (ANDISORD), several tests were executed. Due to space limitations, only some representative results are presented here.

Test 1. A set of six problems: three involving haze, two related to thicker clouds and one concerned with a more realistic atmosphere (including aerosols, etc.) were proposed to the scientific community by the Radiation Commission [11]. ANDISORD was used to solve the first two cases. Results from "case 2" follow.

The characteristics of the problem are: homogeneous plane-parallel atmosphere with total thickness $\tau = 1$ and albedo for single scattering $\omega_o = 0.9$; black, non-emitting ground; incident solar beam from direction $(\mu_o,0) = (-1,0)$ and flux $\pi F = \pi$ $(F = 1)$; anisotropic scattering with the coefficients for the expansion of the phase function into Legendre polynomials given.

Tables 1 and 2 present the result of the computations for comparison with other methods.

In all cases, ANDISORD was run with only six layers (τ = 0.05, 0.05, 0.1, 0.3, 0.25, 0.25) and with 32 streams (16 up and 16 down), while the phase function was approximated with a series of 31 Legendre polynomials (some of the other programs used up to 50) of orders 1 to 31.

ANDISORD provided excellent results. The error in the integrated quantities (fluxes) was always less than 0.1 percent, while for the discretized intensities the maximum error was in the order of 4 percent. Increasing the number of layers and streams would diminish these errors further.

Table 1. Radiance * 10: Haze L

τ	μ	spher. harmo.	Discr. Ord.	FN method	Monte Carlo	Current ANDISORD
0	1	0.2788	0.2784	0.2795	0.283	0.2665
	0.8	0.3139	0.3143	0.3144	0.323	0.3124
	0.2	0.6701	0.6702	0.6696	0.703	0.6715
	0	0.5196	0.5177	0.5175	0.475	0.5255
0.5	1	0.1371	0.1369	0.1374	0.136	0.1317
	0.8	0.1607	0.1609	0.1609	0.163	0.1599
	0.2	0.6676	0.6673	0.6671	0.680	0.6845
	0	0.9410	0.9399	0.9401	1.13	0.9283
	-0.2	0.9168	0.9155	0.9160	0.948	0.9208
	-0.8	2.385	2.385	2.385	2.38	2.3854
	-1	22.48	22.40	22.40	22.4	22.492
1.0	0	0.7962	0.7991	0.7932	0.749	0.8022
	-0.2	1.243	1.242	1.242	1.30	1.2425
	-0.8	3.870	3.870	3.869	3.91	3.8730
	-1	29.77	29.67	29.67	29.8	29.791

Table 2. Flux Case 2: Haze L.

Method	Diffuse Flux		Net Flux			
	$F^+(0)$	$F^-(\tau 1)$	τ=0	τ=0.1	τ=0.5	τ=1
Spheric harm. D	0.1236	0.1516	3.0180	2.9832	2.8418	2.6712
Discrete Ord. L	0.1243	1.537	3.0178	---	---	2.6714
FN Method	0.1237	1.5155	3.0179	2.9831	2.8418	2.6713
Doubling	0.1237	1.5155	3.0179	2.9831	2.8418	2.6713
Finite differ.	0.1233	1.1557	3.0182	2.9835	2.8425	2.6714
Monte Carlo.P	0.1230	1.516	3.019	2.985	2.837	2.672
Delta Eddi. W	0.1471	1.4998	2.9945	2.9601	2.8225	2.6555
2Stream Standard	0.0999	---	3.0417	3.0066	2.6569	---
"exact" value	---	---	3.018	2.9832	2.8418	2.6713
ANDISORD	0.1235	1.5190	3.0181	2.9834	2.8439	2.6748

Test 2. The ability of the program to handle different angles of incidence for the solar beam was tested by evaluation of reflection and transmission (direct plus diffuse) for two "atmospheres" with optical thicknesses of 0.25 and 1.0 respectively. Non conservative scattering (ω_o = 0.8) and Henyey-Greenstein scattering with asymmetry factor (g) of 0.75 was considered. In Table 3 results are compared with those tabulated in Liou [20]. These latest results were obtained with the use of a Discrete Ordinates Method (DOM) with 2, 4, 8, and 16 streams and by the Doubling Method. ANDISORD was run with 16 streams and 10 layers for optical thickness of 0.25, and 16 streams and 16 layers for the optical thickness of 1.0.

Table 3. Reflection and Transmission

		Reflection		Transmission	
τ_1	$\mu_o \Rightarrow$ Method⇓	0.1	0.9	0.1	0.9
0.25	DOM, 2	0.31802	-0.01125	0.46566	0.95403
	4	0.30269	0.01746	0.46032	0.92623
	8	0.29599	0.01473	0.44354	0.92728
	16	0.29406	0.01558	0.43120	0.92679
	Doubling	0.28961	0.01547	0.43017	0.92669
	ANDISORD	0.2897	0.0155	0.4310	0.9267
1.00	DOM, 2	0.37519	-0.00064	0.29023	0.76333
	4	0.37646	0.05425	0.22724	0.72003
	8	0.36938	0.04901	0.20192	0.71702
	16	0.36071	0.04942	0.20416	0.71784
	Doubling	0.35487	0.04929	0.20556	0.71772
	ANDISORD	0.3559	0.0493	0.2073	0.7179

Test 3. The purpose of this test was to compare the results provided by ANDISORD with those from the "exact" zone method in an anisotropicaly scattering media [21,22]. The problem involves a layer of total optical thickness (τ), black walls, conservative scattering, and linearly anisotropic media with the phase function $P(\theta) = 1 + 3g\theta$. Results are given in table 4.

Table 4. Hemispherical Reflectivity of the Slab

		τ		
g	References	0.1	1.0	10.0
-1.0	Byun & Smith [21]	0.1048	0.5138	0.9092
	ANDISORD	0.1061	0.5138	0.9096
-0.7	Dayan & Tien [23]	0.099	0.495	--
	Yuen & Tien [24]	0.099	0.495	--
	Byun & Smith	0.0987	0.4954	0.9026
	ANDISORD	0.1001	0.4954	0.9030
0.0	Bleach et. al [25]	0.0843	0.4466	0.8833
	Busbrige et. al [26]	--	0.4466	0.8833
	Dayan et al	0.084	0.447	0.891
	Ozisik & Yener [27]	--	0.4466	--
	Sutton & Ozisik [28]	--	--	--
	Byun & Smith	0.0843	0.4465	0.8828
	Fiveland (S-6) [29]	--	0.4475	0.8842
	ANDISORD	0.0857	0.4465	0.8832
0.7	Dayan et al	0.069	0.389	--
	Byun & Smith	0.0694	0.3872	0.8529
	ANDISORD	0.0708	0.3872	0.8533
1.0	Busbrige et. al.	--	0.3577	0.8351
	Byun & Smith	0.0628	0.3577	0.8348
	Fiveland (S-6)	--	0.3583	0.8330
	ANDISORD	0.0643	0.3576	0.8351

Test 4. Figure 3 shows the excellent agreement between the dimensionless blackbody emissive power results obtained using ANDISORD and the zone method [22] for the solution of an absorbing, emitting and isotropically scattering medium enclosed by black walls. In this problem, the equation:

$$(-\nabla \cdot q_r)_p = a\left(\sum_{j=1}^{n} w_j I_j - 4\pi I_b^p\right) \tag{12}$$

is solved for $(-\nabla \cdot q_r)_p = 0.0$ (radiative equilibrium)

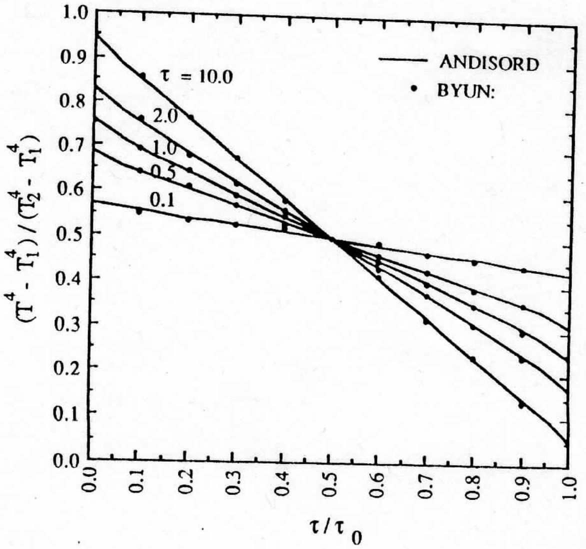

Figure 3. Dimensionless Emissive Power.

Test 5. Wiscombe et al.[30] (see also [31]) provide a series of tests for their computer program DISORD. Some of these tests were run to bench mark ANDISORD. The different "cases" are named as in the original reference.

Table 5 compares results for cases 8A to 8C. These cases involve two inhomogeneous layers (denoted by the subindexes 01 and 02). The source of energy is isotropic radiation of intensity XISO=0.31831 incident on the upper surface and the scattering is isotropic.

TABLE 5. Intensities for two inhomogeneous layers.

						Intensities at μ =	
	At τ =	τ_{01}	τ_{02}	ω_{01}	ω_{02}	-1.0	1.0
	Case 8A						
Method		0.25	0.25	0.5	0.3		
DISORD	0.00					0.31831	0.019442
ANDISORD						0.31830	0.019472
DISORD	0.25					0.262711	0.005519
ANDISORD						0.262728	0.005514
DISORD	0.50					0.210014	0.000000
ANDISORD						0.209978	0.000000
	Case 8B						
		0.25	0.25	0.8	0.95		
DISORD	0.00					0.318310	0.049558
ANDISORD						0.31830	0.049542
DISORD	0.25					0.277499	0.025058
ANDISORD						0.277265	0.024978
DISORD	0.50					0.240731	0.000000
ANDISORD						0.240736	0.000000
	Case 8C						
		1.00	2.00	0.8	0.95		
DISORD	0.00					0.318310	0.104766
ANDISORD						0.31830	0.104748
DISORD	1.00					0.189020	0.065445
ANDISORD						0.188999	0.065383
DISORD	3.00					0.068476	0.000000
ANDISORD						0.068384	0.000000

TABLE 6. Fluxes for two inhomogeneous layers.

						Fluxes	
	At τ =	τ_{01}	τ_{02}	ω_{01}	ω_{02}	Down diffuse	Up diffuse
	Case 8A						
Method		0.25	0.25	0.5	0.3		
DISORD	0.00					1.000000	0.092963
ANDISORD						0.99997	0.093183
DISORD	0.25					0.722235	0.027895
ANDISORD						0.72122	0.027907
DISORD	0.50					0.513132	9.09E-18
ANDISORD						0.51214	0.00000
	Case 8B						
		0.25	0.25	0.8	0.95		
DISORD	0.00					1.000000	0.225136
ANDISORD						0.99997	0.22516
DISORD	0.25					0.795332	0.126349
ANDISORD						0.79494	0.12627
DISORD	0.50					0.650417	2.20E-16
ANDISORD						0.65010	0.00000
	Case 8C						
		1.00	2.00	0.8	0.95		
DISORD	0.00					1.000000	0.378578
ANDISORD						0.99997	0.37849
DISORD	1.00					0.486157	0.243397
ANDISORD						0.48580	0.24319
DISORD	3.00					0.159984	1.19E-17
ANDISORD						0.15972	0.00000

CONCLUSIONS AND RECOMMENDATIONS

Development of a 3D radiation transfer model accounting for hydrometeor scattering, real gas participation and all the boundary conditions appropriate for problems related to atmospheric radiation and satellite rainfall retrieval is an important objective of satellite remote sensing.

As a first step in the fulfillment of the goal, a 1D computer program (ANDISORD) was developed and implemented on a personal computer. The program, based on the finite difference version of the discrete ordinates method, was tested in a variety of situations with excellent results.

Although the program was not timed, it can be said that in all the cases tested, the required wall clock time was only a few seconds.

Future testing of ANDISORD, in its 1D version, as well as POLY – the companion program for the determination of the optical properties of polydispersions– will include thicker atmospheres and interaction with conduction and convective heat transfer. These are applications for which the program is already designed. In the near future, the 1D version will be enhanced to account for non-lambertian ground.

In addition to atmospheric remote sensing, the program can be readily applied to a wide variety of engineering problems. A recently developed 2D version of the program was applied to the problem of combined natural convection and radiation in a rectangular enclosure [14].

ANDISORD is available, upon request, to qualified users.

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