VIEW FACTORS OF A RECTANGULAR SYSTEM ENCLOSING A TRANSPARENT MEDIA BY DISCRETE-ORDINATES METHOD

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ABSTRACT

The purpose of this study is to compute the view factors for a rectangular system by the discrete- ordinates method. The effects of number of spatial divisions, number of discrete ordinates directions, and the values of the weighting factors on the accuracy of the solution are studied. If shading appears, the accuracy of the solution depends both on the number of discrete ordinates directions and number of spatial divisions. Three different weighting factor sets, equal weighting factors for each direction cosine, Chevyshef quadratures, and weighting factors from trapezoidal integration rule are tested. The results are not sensitive to the choice of the three different weighting factor sets.

NOMENCLATURE

E_b	blackbody emissive power
F_{S-N}	view factor from south wall to north wall
H, H_1	height of system and protrusion
I	intensity
L, L_1	width of system and protrusion
M	number of division for one quadrant
n	number of spatial division
q	radiative heat flux, W/m^2
T	temperature, K

Greeks

α	finite difference weighting factor
η_i, μ_i	y-direction cosine, x-direction cosine
σ	Stefan-Boltzmann coefficient
ω_i	i-discrete ordinate weighting factor

Subscripts

b	blackbody
N,S,E,W	north, south, east, west
i	i-discrete ordinate direction

INTRODUCTION

There are many applications of surface to surface radiative heat exchange in transparent media such as heating and cooling of raw or finished materials, drying, heat transfer in rooms, and cooling of electronic components. Many kinds of methods appear in the literature to analyze radiative heat exchange in these systems. For example, the method of using the view factor (Hottel and Sarofim, 1967), cross string method (Hottel and Sarofim, 1967 ; Siegel and Howell, 1981), Monte Carlo method (Siegel and Howell, 1981), and Stochastic method (Naraghi and Chung, 1984) are used.

View factor results for various geometry continue to be published by many researchers. View factors can be calculated by direct integration, Monte Carlo method (Siegel and Howell, 1981), and ray tracing method (Baumeister, 1990). Recently, Sanchez and Smith (1992) proposed that view factors can be obtained by the discrete-ordinates method. The discrete-ordinates method is used for the analysis of radiative heat transfer where participating media appear (Chandrasekhar, 1950; Fiveland, 1984,1988; Kim and Lee, 1988; Kim and Baek, 1991; Truelove, 1987).

Sanchez and Smith (1992) published the radiative heat transfer results for a two-dimentional rectangular system enclosing a transparent medium. The system may have protrusions or inserted rectangular material. The effects of finite-difference weighting factor, grid pattern, number of discrete ordinates angle, effect of surface emittance are studied. However, the results for the view factors are not published.

The primary purpose of this study is to find the view factors of two dimensional rectangular system by using the discrete-ordinates method. The secondary purpose of this study is to find the value of the finite difference weighting factor that certifies the exact solutions as when increasing the number of spatial and angular divisions. It is known that the finite-difference weighting factor (Fiveland, 1984,1988; Kim and Lee, 1988; Kim and Baek, 1991; Truelove, 1987) has a meaning when analyzing the system enclosing a participating media. The question is whether the finite difference weighting factor is necessary for analyzing a system enclosing a transparent media. Thirdly, to study the effect of the different sets of discrete-ordinates weighting factors on the accuracy of the results.

ANALYSIS

GOVERNING EQUATIONS

The system under study is described as in Fig. 1 with the coordinate system.

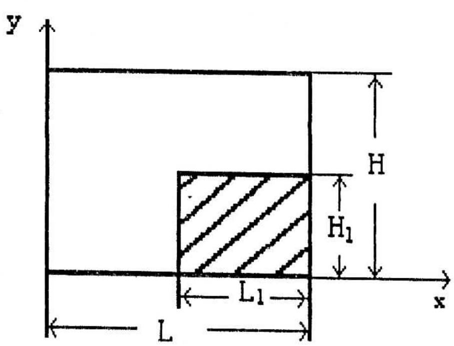

Fig. 1. System description

The bounary surfaces are opaque and black. The medium in the system is nonparticipating. It is assumed that the heat transfer occurs only by thermal radiation. The length and height of the system are L, H , respectively. The protrusion length and heights are L_1, H_1, respectively.

The arbitrary control volume that appears in Fig. 2 is generated by dividing the length by n_j and height by n_i. The center point of the control volume is designated as P. Subscripts N,S,E,W appear on the north, south, east, and west surfaces with respect to the point P, respectively.

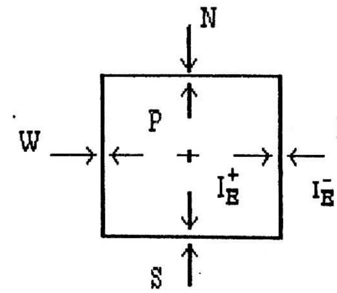

Fig. 2. Control volume description

A positive superscript (+) is assigned for the intensity going out from the control volume and a negative superscript (-) is assigned for the intensity entering the control volume. The rotation angle 2 π is equally divided at the point P. In this paper, each of 4 quadrants is divided by M angular directions, thus, there are 4 M discrete ordinates directions.

The radiative transport equations for an arbitrary i-discrete direction is

$$\frac{dI_i}{ds} = \mu_i \frac{\partial I_i}{\partial x} + \eta_i \frac{\partial I_i}{\partial y} = 0 \qquad (1)$$

I is the intensity, and s is the length of travel. μ_i and η_i is x- and y-direction cosines.

For transparent medium, the divergence of radiative flux for i-discrete direction is expressed as

$$\frac{\partial q_{ix}}{\partial x} + \frac{\partial q_{iy}}{\partial y} = 0 \qquad (2)$$

where q_{ix} and q_{iy} are the radiative flux in the x- and y-direction , respectively. The radiative fluxes expressed in terms of intensity are,

$$q_{ix} = \mu_i I_i \ , \ q_{iy} = \eta_i I_i \qquad (3)$$

If Eq. (1) is discretized by using the intensity at the point P and the intensities at the west and south boundaries, then Eq. (1) is transformed to

$$\mu_i (I^+_{iP} - I^-_{iW}) \Delta y + \eta_i (I^+_{iP} - I^-_{iS}) \Delta x = 0 \qquad (4)$$

where Δx and Δy are the control volume length and height, respectively. In case $\Delta x = \Delta y$, Eq. (4) reduces to

$$I^+_{iP} = \frac{\mu_i I^-_{iW} + \eta_i I^-_{iS}}{\mu_i + \eta_i} = \frac{\mu_i I^-_{iE} + \eta_i I^-_{iN}}{\mu_i + \eta_i} \qquad (5)$$

The last term in Eq. (5) is obtained by discretizing Eq. (1) by using the intensity at the point P and the intensities at the east and north boundaries. If Eq. (2) is discretized by utilizing Eq.(3) and the P,W,S,E,N intensities, the resulting equation is same as Eq.(5). Thus, Eq. (5) satisfies both Eqs. (1) and (2).
If the radiative heat flux at the east surface is expanded about the radiative heat flux at the point P by Taylor series , then

$$q_{iE} = q_{iP} + \frac{\partial q_{iP}}{\partial x}\frac{\Delta x}{2} \tag{6}$$

After spatially discretizing the differential terms in Eq. (6) by using q_E and q_W, Eq. (3) is substituted into Eq.(6). The result is

$$I^+_{iE} = I^+_{iP} + \frac{I^+_{iE} - I^-_{iW}}{2} = 2\,I^+_{ip} - I^-_{iW} \tag{7a}$$

The following equations are obtained by using a similar procedure as described for Eq. (7a).

$$I^+_{iN} = 2\,I^+_{iP} - I^-_{iS} \tag{7b}$$

$$I^+_{iW} = 2\,I^+_{iP} - I^-_{iE} \tag{7c}$$

$$I^+_{iS} = 2\,I^+_{iP} - I^-_{iN} \tag{7d}$$

The positive intensities at the point P become the negative intensities at the adjacent control volumes since these are incoming fluxes. The Equation (7) is the same as the intensity relations in the paper by Sanchez and Smith (1992) if the value of the finite difference weighting factor α is 0.5. Thus, as long as $\Delta x = \Delta y$ and the point P is located at the center of the control volume, the value of the finite difference weighting factor α should be 0.5.

BOUNDARY CONDITIONS

If the boundary walls are opaque and black, the radiative intensity of the i-discrete direction is

$$I^-_{i,wall} = \frac{E_{bi,wall}}{\pi} \qquad \text{wall} = N, S, E, W \tag{8}$$

$E_{bi,wall}$ is the black body emissive power of i-discrete direction. The black body emissive power of i- discrete direction is

$$E_{bi} = \frac{E_b}{4M} = \frac{\sigma T^4}{4M} \tag{9}$$

where σ is the Stefan-Boltzmann constant, and T is the absolute temperature, and M is the number of discrete-ordinates direction of each quadrant.

THE RADIATIVE HEAT FLUX

The radiative flux q_{wall} at each boundary surfaces are as follows.

$$q_{wall} = \sum_{i=1}^{2M} \mu_i\,\omega_i (I^+_i - I^-_i) \qquad \text{wall} = E, W \tag{10a}$$

$$q_{wall} = \sum_{i=1}^{2M} \eta_i\,\omega_i (I^+_i - I^-_i) \qquad \text{wall} = N, S \tag{10b}$$

where ω_i is the weighting factor for i-discrete-ordinates direction.

DISCRETE ORDINATE DIRECTIONS AND WEIGHTING FACTORS

The Method by Sanchez and Smith (1992)

The discrete ordinate direction angles are generated by dividing the quadrant polar angle $\pi/2$ by M.

$$\phi_i = \frac{\Delta\phi}{2} + (i-1)\Delta\phi, \qquad \Delta\phi = \frac{\pi}{2M} \qquad i=1,M \tag{11}$$

By assuming equal weighting factor for each direction, the values of weighting factors are

$$\omega = \frac{\pi}{2} \Big/ \sum_{i=1}^{M} \cos\phi_i \tag{12}$$

Chevyshef Quadrature

This set is also based on equal weights and an equal angular increments. Discrete angles are the same as in Eq. (11). The weighting factor of Chevyshef quadrature (Atkinson, 1978) is obtained by dividing the quadrant polar angle $\pi/2$ by M.

$$\omega = \frac{\pi}{2} \tag{13}$$

Trapezodial Method

Discrete angles have an equal increment defined by

$$\phi_i = (i-1)\Delta\phi, \qquad \Delta\phi = \frac{\pi}{2M} \qquad i=1,M+1 \tag{14}$$

There are M+1 discrete ordinate directions including two of quadrant boundary directions. The values of the weighting factors used for this case are as follows (Atkinson, 1978). Equal weights for the inner direction weighting factors. The weighting factors for the boundary directions are the half of the inner weighting factors.

$$\omega_i = \frac{\pi}{2M} \qquad i = 2, M$$

$$\omega_i = \frac{\pi}{4M} \qquad i = 1 \text{ or } M+1 \qquad (15)$$

NUMERICAL SOLUTION PROCEDURE

It is assumed that entering intensities at the system boundary walls are defined.

[1] Starting from the control volume at the corner of the system boundary, choose one arbitrary discrete ordinate direction.

[2] Compute the intensities leaving the control volume by using Eqs. (5) and (7). The intensities leaving the control volume become the intensities entering two neighboring control volumes. Continue this procedure until all the control volumes are swept.

[3] Repeat the steps from [1] to[2] , starting from remaining three corner control volumes.

[4] At each control volume, find value of the radiative flux by Eq.(3). This step may be done at step [6] if there are no storage problems.

[5] Repeat the steps from [1] to [4] for all the remaining discrete ordinate directions.

[6] By using Eq.(10), compute the radiative flux at each surfaces.

Table 1. Effects of n and M on view factor F_{S-N}

[AJ : (Sanchez and Smith, 1992), CH: Chevyshef weight, TP: Trapezoidal method]

		M				
n	Method	2	4	8	64	512
2	AJ	0.31371	0.36020	0.36312	0.36259	0.36258
	CH	0.31371	0.36020	0.36312	0.36259	0.36258
	TP	0.41421	0.36197	0.36108	0.36255	0.36258
8	AJ	0.39769	0.42256	0.41205	0.41140	0.41141
	CH	0.39769	0.42256	0.41205	0.41140	0.41141
	TP	0.41421	0.40562	0.41417	0.41142	0.41141
16	AJ	0.40792	0.41250	0.41779	0.41345	0.41352
	CH	0.40792	0.41250	0.41779	0.41345	0.41352
	TP	0.41421	0.41094	0.41173	0.41362	0.41352
32	AJ	0.41376	0.41305	0.41317	0.41400	0.41404
	CH	0.41376	0.41305	0.41317	0.41396	0.41404
	TP	0.41421	0.41398	0.41351	0.41417	0.41404
64	AJ	0.41495	0.41354	0.41394	0.41417	0.41417
	CH	0.41495	0.41354	0.41394	0.41417	0.41417
	TP	0.41421	0.41460	0.41406	0.41419	0.41417

RESULTS AND DISCUSSION

The method described in the analysis is applied to two examples. For the first example, the method is applied to compute the view factors for a square geometry without the protrusion. The purpose of this example is to show the accuracy of the method because the exact values are available. The emissive power of south surface (S) is assigned as 1, and the emissive powers of the other surfaces are zero. The radiative flux at each surfaces is the value of the view factors from south surface (S) to the target surface (Sanchez and Smith, 1992). The boundary surfaces are divided by n. The effect of number of the discrete-ordinates method (DOM) weighting factors on F_{S-N} are presented in Table 1.

The value of view factor F_{S-E} and F_{S-W} should be same due to symmetry. Equality of F_{S-E} and F_{S-W} are checked. Also, due to enclosure relation (Hottel and Sarofim, 1967), $F_{S-E} = (1 - F_{S-N}) / 2$. Thus, only the the view factor from south wall to north wall, F_{S-N}, appears in Table 1. The five significant digit exact value of F_{S-N} is equal to 0.41421.

In Table 1, the value of the view factor by discrete ordinate method converges to the exact value as n and M increases. For n = 8 or 16, if $M \geq 8$ then the relative error is within $\pm$ 1 % else within $\pm$ 5 %. If $n \geq 32$, for any M, the relative error is within $\pm$ 1 %. Thus, the accuracy of the solution depends more on the number of spatial divisions than the number of discrete ordinate directions. Even at M=2, the vaules of the view factors by the Trapezoidal Method are accurate. However, when the energy balance and surface radiative flux values are checked for this case, some errors are found.

The radiative fluxes at the north and the east wall are predicted by several methods. The results are compared in Table 2. Height and width are equally divided by n = 60 (Sanchez and Smith, 1992). The temperatures of the boundary walls are T_S = 310 K, and $T_N = T_E = T_W$ =300 K. All of the boundary walls are black opaque surfaces. In this case, the radiative flux at the east wall is equal to the west wall.

The wall radiative flux values are computed by using the weighting factors as in Refs. (Sanchez and Smith, 1992; Fiveland, 1984,1988; Truelove, 1987) and the results are tabulated in Table 2. The weighting factors in Refs. (Fiveland, 1984, 1988; Truelove, 1984) are used for computing the radiative heat transfer by the participating media enclosed in a three dimensional rectangular box geometries.

Table 2. Overall heat fluxes

(1) Norm wall heat flux, W/m^2

M	Sanchez & Smith (1992)	Fiveland (1988)	Truelove (1987)	Fiveland (1984)	Chevy -shef	Trape- zoidal
2	26.654*				27.296	25.272
3	23.576*	26.235*	23.518*	23.518*	23.848	29.132
4	26.717*				26.835	26.284
6	26.676*	25.497*	23.366*		26.752	26.490
10	26.667	22.487*			26.722	26.576
15	26.687				26.528	26.723
20	26.708 (26.603)				26.607	26.702
50	26.727 (26.656)*				26.657	26.655
512	26.653*				26.654	26.654
RIM (Sanchez & Smith, 1992)						26.657

(2) East wall heat flux, W/m^2

M	Sanchez. & Smith (1992)	Fiveland (1988)	Truelove (1987)	Fiveland (1984)	Chevy -shef	Trape- zoidal
2	18.850*				19.372	17.870
3	20.390*	19.060*	21.461*	21.461*	20.624	16.873
4	18.819*				18.968	18.621
6	18.840*	19.429*	21.333*		18.894	18.749
10	18.844	20.934*			18.850	18.823
15	18.834				18.928	18.787
20	18.824				18.853	18.837
25	18.820 (18.876)				18.879	18.816
50	18.814 (18.849)*				18.850	18.848
512	18.851*				18.851	18.851
RIM (Sanchez & Smith, 1992)						18.849

Numbers in parentheses are for α = 0.5. (Sanchez and Smith, 1992). Numbers with * are results of authors program.

The results in Table 2 are the results of the code written by the authors. Only the values of the weighting factors in Refs , (Fiveland, 1984,1988; Truelove, 1987) are used for the results. The value of the finite difference weighting factor α is 0.5.

The number of directions per quadrant for S4, S6, S8 discrete ordinate method is M = 3, 6, 10, respectively. α = 0.6 is utilized for the results of Sanchez and Smith (1992) in Table 2 except the results enclosed by parenthesis where α = 0.5 is used. It can be observed from the results presented in Table 2 that if $\Delta x = \Delta y$ and the point P in Fig. 2 is located at the center of the control volume , the value of finite difference weighting factor should be $\alpha = 0.5$ to obtain exact values by increasing the number of discrete angles.

In example 2, the DOM is applied to find the view factors of the system that has a protrusion as in Fig. 1. The boundary surfaces are opaque and black, and $H_1/H = 0.5$, $L_1/L = 0.5$. The emissive power at the south wall is 1. The emissive powers at the other five surfaces are 0. There is shading from south wall to east wall and to north wall. The width and height of the enclosure are equally divided by n. In Table 3, the effect of the number of discrete directions on the view factor from the south to north wall, $F_{S\text{-}N}$,is studied. The exact five digit value of $F_{S\text{-}N}$ is 0.32514. The view factor results by the Chevyshef quadrature are same as the results presented in Table 3 of Sanchez and Smith (1992), and thus, the results are omitted in Table 3.

Table 3. Effects of n and M on view factor $F_{S\text{-}N}$

[AJ: (Sanchez and Smith, 1992), TP: Trapezoidal method]

		M				
n	Method	2	4	8	64	512
2	AJ	0.24264	0.29323	0.30220	0.30448	0.30451
	TP	0.41421	0.32503	0.30898	0.30458	0.30452
8	AJ	0.24032	0.33236	0.31767	0.32124	0.32124
	TP	0.41421	0.32383	0.32813	0.32123	0.32124
16	AJ	0.24982	0.31899	0.32712	0.32408	0.32417
	TP	0.41421	0.32877	0.32383	0.32427	0.32417
32	AJ	0.26147	0.32065	0.32211	0.32483	0.32490
	TP	0.41421	0.33482	0.32767	0.32502	0.32490
64	AJ	0.27046	0.32159	0.32206	0.32505	0.32508
	TP	0.41421	0.33949	0.33045	0.32518	0.32508

In Table 3, the value of the view factor by DOM converges to the exact values as n and M increases. If $n \geq 16$ and M =4, the relative error observed is within $\pm$ 5 %. If $n \geq 16$ and $M \geq 8$, the relative error observed is within $\pm$ 1 %. Therefore, if there is shading, the view factor results by DOM are sensitive both the number of discrete ordinate directions and the number of spatial divisions.

CONCLUSIONS

The purpose of this study is to find the view factors for a two dimensional rectangular system enclosing a transparent media by using the discrete ordinate method. From the examples presented in this paper, conclusions are as follows. First, view factors of the system with or without shadings can be obtained by using the discrete-ordinates method. Second, as long as the shape of a control volume is square and point P in control volume is located at the center, the finite difference weighting factor should be 0.5. If not, then the solution does not converge to the exact values. Third, if there is shading in the system, both the number of spatial divisions and angular divisions have an effect on the accuracy of the solution. Not much differences are found by using the several direction weighting factors from Sanchez and Smith (1992), the Chevyshef quadrature, and Trapezoidal integration rule.

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