APPLICATION OF THE DISCRETE ORDINATES METHOD TO 3-D CLOUD RADIATIVE TRANSFER

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1. INTRODUCTION

Earth's energy balances, global circulation patterns, and climatic variations are examples of studies in which quantitative understanding requires modeling of the three-dimensional aspects of cloud radiation. Proper interpretations of satellite sensed data of radiant intensity for extraction of information related to cloud properties, ground temperature, and rainfall require a radiative transfer model.

Three-dimensional studies of cloud radiative transfer are limited. Studies of radiative properties of broken cloud fields (Stephens 1988; Welch and Wielicki 1989; Kobayashi 1989) are examples. The work cited used the Monte Carlo approach or an extensive mathematical formulation for solving the radiative transfer problem. Modeling of convective clouds and rainfall (Mugnai et al. 1990; Adler et al. 1991) considered three-dimensional cloud structures but only one-dimensional radiative transfer.

The objective of this paper is to apply the discrete-ordinates method to examine three-dimensional cloud radiative transfer. The outgoing intensity for application to remote sensing studies is calculated. Although the real shape of clouds can vary widely, and is best described by fractals (Mandelbrot, 1983), cubical arrays are commonly used (McKee and Cox, 1976; Welch and Wielicki, 1989) to study the radiative characteristics and properties of clouds. The cloud field as illustrated in Fig. 1 consists of an array of cubic clouds of dimensions L_1 separated a distance L_2 with a height L_3. The cloud field is irradiated with direct collimated solar energy. The ground is diffusely reflecting and opaque. Thermal effects are neglected.

2. THE DISCRETE ORDINATES METHOD

Several authors have presented formulations of the discrete-ordinates method (Carlson and Lathrop 1968; Stamnes et al. 1988; Gerstl and Zardecki 1985; Fiveland and Jamaluddin 1989; Kim and Lee 1989). The discretized form of the radiative transfer equation is obtained by subdividing the entire three-dimensional domain into cubical control volumes and discretizing the direction of propagation of the radiant intensity. Typical control volumes are shown in Fig. 2, where control volume P with differential volume $\Delta V = \Delta x \ \Delta y \ \Delta z$ is of interest. Control volume P is surrounded by six adjacent control volumes labeled W (west), E (east), S (south), N (north), F (front), and B (back) with associated interfaces of w, e, s, n, f, and b. Each control volume is homogeneous, and nonhomogeneities are accounted for by assigning different radiative properties to the control volumes. For unpolarized radiation, the radiant intensity in direction i for control volume P is

$$I_i^P = \frac{\frac{|\mu_i|}{\Delta x} I_i^{xr} + \frac{|\delta_i|}{\Delta y} I_i^{yr} + \frac{|\gamma_i|}{\Delta z} I_i^{zr} + \alpha S}{\frac{|\mu_i|}{\Delta x} + \frac{|\delta_i|}{\Delta y} + \frac{|\gamma_i|}{\Delta z} + \alpha \beta} \tag{1}$$

where the direction cosines μ_i, δ_i, and γ_i are for the x-, y-, and z-directions. Superscripts r designate the interface from which the radiant energy originates for the indicated coordinate. The intensities arriving at the end-faces (which become the reference intensities for the neighboring control volumes) are recovered from

$$I_i^{xe} = [(I_i^P + (\alpha-1)\ I_i^{xr}]/\alpha \tag{2}$$

Expressions for I_i^{ye} and I_i^{ze} can be written by replacing x with y and z. The finite-difference weighting factor α is taken as unity. The radiant source S for inward scattering and a collimated source are

$$S = \frac{s}{4\pi} \sum_{j=1}^{K} w_j \ I_j^P \ \Phi_{ij} + \frac{s}{4\pi} \ I_c^P \ \Phi_{ic} \tag{3}$$

In Eqs. (1-3), β is the extinction coefficient, s the scattering coefficient, Φ_{ij} the phase function for scattering between the i and j discrete directions, K the number of discrete directions in a spherical space, and w_j the quadrature weight for the j direction. The direct intensity I_c^P at point P is

$$I_c^P = I_c \exp(- \int_0^{\zeta} \beta \ d\zeta^*) \tag{4}$$

where ζ is the location of point P and I_c the direct solar intensity at the boundary from direction $\bar{\omega}_c$. The sensed intensity as measured by a radiant sensor placed at P is the sum of the intensity from Eq. (1) plus the direct intensity from Eq. (4), recognizing that the direct intensity is only added when the sensed intensity is sought for the direction $\bar{\omega} = \bar{\omega}_c$.

The boundary condition for Eq. (1) is that the intensity is zero at all boundaries except along the ground surface where it is given by

$$I_i^+ = \frac{\rho_d}{\pi} \left[I_c^- + \sum_{j=1}^{K/2} w_j \ \eta_j \ I_j^- \right] \tag{5}$$

η_i is the cosine of the angle between the normal to the surface (boundary) and the direction of propagation j. The summation in Eq. (5) is over