Mattis, D. C., and Bardeen, J., 1958, "Theory of the Anomalous Skin Effect in Normal and Superconducting Metals," *Phys. Rev.*, Vol. 111, pp. 412–417.

Miller, D., Richards, P. L., Etemad, S., Inam, A., Venkatesan, T., Dutta, B., Wu, X. D., Eom, C. B., Geballe, T. H., Newman, N., and Cole, B. F., 1992, "Correspondence Between Microwave and Submillimeter Absorptivity in Epitaxial Thin Films of $YBa_2Cu_3O_7$," *Phys. Rev. B*, Vol. 47, pp. 8076–8088.

Pham, T., Lee, M. W., Drew, H. D., Welp, U., and Fang, Y., 1991, "Far-Infrared Absorptivity of Single-Domain $YBa_2Cu_3O_7$," *Phys. Rev. B*, Vol. 44, pp. 5377–5381.

Phelan, P. E., Flik, M. I., and Tien, C. L., 1991, "Radiative Properties of Superconducting Y-Ba-Cu-O Thin Films," ASME JOURNAL OF HEAT TRANSFER, Vol. 113, pp. 487–493.

Piel, H., and Müller, G., 1991, "The Microwave Surface Impedance of High-T_c Superconductors," *IEEE Trans. Magn.*, Vol. 27, pp. 854–862.

Press, W. H., Flannery, B. P., Teukolsky, S. A., and Vetterling, W. T., 1986, *Numerical Recipes*, Cambridge University Press, Cambridge, UK, Chap. 14.

Renk, K. F., Gorshunov, B., Schützmann, J., Prückl, A., Brunner, B., Betz, J., Orbach, S., Klein, N., Müller, G., and Piel, H., 1991, "Far-Infrared and Microwave Intragap Absorption in a $YBa_2Cu_3O_{7-\delta}$ Thin Film at Low Temperature," *Europhys. Lett.*, Vol. 15, pp. 661–666.

Scharnberg, K., 1978, "Effect of Finite Electron Mean Free Path on the Attenuation, Electromagnetic Generation, and Detection of Ultrasonic Shear Waves in Superconductors," *J. Low Temp. Phys.*, Vol. 30, pp. 229–263.

Schlesinger, Z., Collins, R. T., Holtzberg, F., Feild, C., Blanton, S. H., Welp, U., Crabtree, G. W., Fang, Y., and Liu, J. Z., 1990, "Superconducting Energy Gap and Normal-State Conductivity of a Single-Domain $YBa_2Cu_3O_7$ Crystal," *Phys. Rev. Lett.*, Vol. 65, pp. 801–804.

Schützmann, J., Ose, W., Keller, J., Renk, K. F., Roas, B., Schultz, L., and Saemann-Ischenko, G., 1989, "Far-Infrared Reflectivity and Dynamical Conductivity of an Epitaxial $YBa_2Cu_3O_{7-\delta}$ Thin Film," *Europhys. Lett.*, Vol. 8, pp. 679–684.

Timusk, T., and Tanner, D. B., 1989, "Infrared Properties of High-T_c Superconductors," in: *Physical Properties of High-Temperature Superconductors*, D. M. Ginsberg, ed., World Scientific Publishing Co., Singapore, Vol. 1, pp. 339–407.

Toscano, W. M., and Cravalho, E. G., 1976, "Thermal Radiation Properties of the Noble Metals at Cryogenic Temperatures," ASME JOURNAL OF HEAT TRANSFER, Vol. 98, pp. 438–445.

Van der Marel, D., Habermeier, H.-U., Heitmann, D., König, W., and Wittlin, A., 1991, "Infrared Study of the Superconducting Phase Transition in $YBa_2Cu_3O_{7-x}$," *Physica C*, Vol. 176, pp. 1–18.

Zhang, Z. M., Choi, B. I., Le, T. A., Flik, M. I., Siegal, M. P., and Phillips, J. M., 1992, "Infrared Refractive Index of Thin $YBa_2Cu_3O_7$ Superconducting Films," ASME JOURNAL OF HEAT TRANSFER, Vol. 114, pp. 644–652.

Zimmermann, W., Brandt, E. H., Bauer, M., Seider, E., and Genzel, L., 1991, "Optical Conductivity of BCS Superconductors With Arbitrary Purity," *Physica C*, Vol. 183, pp. 99–104.

Dimensionality Issues in Modeling With the Discrete-Ordinates Method

A. Sánchez A.,[1] T. F. Smith,[2] and W. F. Krajewski[3]

Nomenclature

a = absorption coefficient, m^{-1}
I = radiation intensity, W/m^2-sr-μm
L = length, m
s = scattering coefficient, m^{-1}
S = source term, W/m^3-sr-μm
x, y, z = coordinates, m
$\Delta x, \Delta y, \Delta z$ = differential lengths in x, y, and z directions, m
α = finite-difference weighting factor
β = extinction coefficient $= (a+s)$, m^{-1}
θ = polar angle, rad
μ, δ, γ = cosine of angles between I and x, y, and z axes
τ = optical thickness $= \beta L$
ϕ = azimuthal angle, rad

Subscripts

c = parallel beam
d = sensor
i = discrete direction i
x, y, z = coordinates

Superscripts

P = control volume label
xr, yr, zr = reference faces in x, y, and z directions

Introduction

The discrete-ordinates method has been successfully applied to the solution of one-, two-, and three-dimensional radiative transfer problems in Cartesian coordinates (Carlson and Lathrop, 1968; Gerstl and Zardecki, 1985; Stamnes et al., 1988; Kim and Lee, 1990; Fiveland and Jamaluddin, 1991; Sánchez et al., 1991). Models of the discrete-ordinates method have been developed to take advantage of the symmetry and invariance related to each level of dimensionality. These models fail to retain, in one- and two-dimensional applications, the three-dimensional effects implied by the presence of incident collimated sources and/or detecting sensors at locations that are outside the planes of symmetry. As a consequence, most applications found in the literature limit parallel beam radiation and intensity output to a sensor to zero azimuthal angles (Kim and Lee, 1989) or a further treatment of the intensity field is required (Stamnes et al., 1988).

In a few occasions, a higher dimensionality (three-dimensional) discrete-ordinates model has been used to solve lower dimensionality (one- or two-dimensional) problems by (1) modifying the aspect ratio of the numerical domain to imply infinite length(s), (2) modeling the boundaries in the infinite direction(s) as specular boundaries, or (3) assuming the boundary conditions in the infinite direction(s) periodic. This classical approach is not practical in optically thick problems where a very large number of control volumes, and, therefore, computer resources, would be required. Because of this and other disadvantages of the classical methodology (ray and end effects), the mirror technique (periodic or symmetric boundaries) was introduced by Sánchez et al. (1991).

After each iteration for the control volume intensity, the discrete-ordinates method gives the intensity field. A mirror technique can be applied to the direction (two-dimensional) or directions (one-dimensional) of infinite length making the boundaries in these directions behave as periodic boundaries. Thus, the intensities leaving a surface on the boundary are placed as intensities arriving at the opposite boundary for the next iteration for the infinite direction(s). In the mirror technique, the aspect ratio is irrelevant and no special consideration about the side walls is needed. Although the mirror approach is a step in the right direction, its slow rate of convergence reduces its practical applicability.

The two approaches (classical and mirror) to the lesser dimensionality problem fail to recognize the simple solution offered by the discrete-ordinates formulation. The verification of a simple procedure (shortcut approach) that uses the same

[1]Professor, Escuela de Ingeniería Mechánica, Universidad de los Andes, Mérida, Venezuela.
[2]Professor, Department of Mechanical Engineering, The University of Iowa, Iowa City, IA 52242; Mem ASME.
[3]Associate Professor, Department of Civil and Environmental Engineering, Iowa Institute of Hydraulic Research, The University of Iowa, Iowa City, IA 52242.

Contributed by the Heat Transfer Division and based on a paper presented at the National Heat Transfer Conference, San Diego, California, August 9–12, 1992. Manuscript received by the Heat Transfer Division May 1992; revision received March 1993. Keywords: Numerical Methods, Radiation. Associate Technical Editor: W. A. Fiveland.

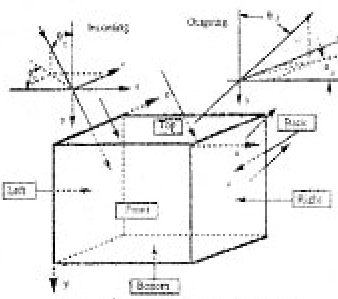

Fig. 1 General three-dimensional geometry

Table 1 Heat flux for receiving wall

a. Average values		
Siegel and Howell (1981)	30417.00	W/m^2
Fiveland and Jamaluddin (1991)	29991.00	W/m^2
ANDISORD4 (Classical approach)	29991.75	W/m^2
b. Center values		
Fiveland and Jamaluddin (1991)	>31000.00	W/m^2
ANDISORD4 (Classical approach)	30483.38	W/m^2
ANDISORD4 (Mirror approach)	30576.84	W/m^2
ANDISORD4 (Shortcut approach)	30572.24	W/m^2

three-dimensional code, with the same quadrature set and without modifications, for the solution of one- and two-dimensional problems is the objective of this study. This approach allows the comparisons and parameterizations of predictions obtained using different dimensionalities in the solution of a given problem to be free from the influences of different quadratures or solution procedures.

Analysis

The three-dimensional system is shown in Fig. 1. The media, which can absorb, emit, and scatter radiation, is contained in a parallelepiped whose walls can be, in any combination, opaque or transparent. The direction of incidence of the parallel beam as well as the direction of the sensed intensity are arbitrary for both polar an azimuthal angles. The present formulation for the discrete-ordinates method is based on that of Sánchez et al. (1992) and Haferman et al. (1993). The three-dimensional geometry is discretized spatially into a number of control volumes, each with uniform properties. Of particular interest in this study is the expression for the radiant intensity for a control volume. In terms of the reference intensities, the intensity for control volume P in discrete direction i is given by

$$I_P^i = \frac{\dfrac{\mu_i}{\Delta x} I_x^i + \dfrac{\xi_i}{\Delta y} I_y^i + \dfrac{\eta_i}{\Delta z} I_z^i + \omega S}{\dfrac{\mu_i}{\Delta x} + \dfrac{\xi_i}{\Delta y} + \dfrac{\eta_i}{\Delta z} + \omega\beta} \tag{1}$$

Note that the dimensionality of the problem is controlled by the presence of the direction cosines. For example, if $\mu = \xi = 0$, then a one-dimensional problem in the z direction is under study. Expressions for the reference intensities, the radiant source term, and the radiative flux are available (Sánchez et al., 1992; Haferman et al., 1993). The finite-difference weighting factor α is taken as unity.

The shortcut approach to simulate lower dimensionalities is implemented as follows:

1 A quadrature set is selected (three-dimensional). For the results that follow, the level sequential-odd quadrature (Fiveland, 1991) is used.

2 The scattering phase function is calculated for all pairs of directions (including, if present, beam and sensor). Using the quadrature set, the cosine of the angle between all pairs of discrete directions is found. The phase function is calculated from the scattering for a single sphere, and the integrated phase function for polydispersions is evaluated from standard formulas. The phase function is expressed in terms of Legendre polynomials and is renormalized. The phase function values preserve the implicit three-dimensional information due to beam and/or sensor.

3 The direction cosines in the direction (two-dimensional) or directions (one-dimensional) of infinite length are set to zero.

4 The resulting system of governing equations is solved as usual.

The shortcut approach is incorporated into a three-dimensional discrete-ordinates code, called ANDISORD4 (Sánchez et al., 1992) and allows solution of one-, two-, or three-dimensional geometries independently of the boundary conditions and presence or not of collimated sources and/or sensors.

Results and Discussion

Test 1. Siegel and Howell (1981) and Fiveland and Jamaluddin (1991) predicted radiative transfer between two parallel, nongray plates spaced 2.54 cm apart in the y direction of Fig. 1. The hot and cold plates are maintained at 1111 K and 556 K. Pure CO_2 at a pressure of 1.013 MPa and a temperature of 556 K is between the plates. Spectral wall emittances and gas absorption coefficients are taken from Fiveland and Jamaluddin (1991). Three different approaches to the numerical solution of this one-dimensional problem using ANDISORD4 are presented.

Classical Approach. Siegel and Howell (1981) used analytical approaches to solve the one-dimensional problem and reported the heat flux on the receiving (cold) wall given in Table 1(*a*). Fiveland and Jamaluddin (1991) used a three-dimensional enclosure with 4×4×4 control volumes and with an aspect ratio of 24:1:24 ($L_x/L_y = 24$, and $L_z/L_y = 24$). The end walls are cold, diffusely reflecting surfaces (with an emittance of 0.0001). Fiveland and Jamaluddin (1991) suggested, based on the average heat fluxes presented in Table 1(*a*), that their modeling of the one-dimensional geometry by means of the three-dimensional code was correct and that the nonuniform profiles shown in Fig. 2 are a consequence of end effects, which could be improved by increasing L_x/L_y and L_z/L_y. Using the same approach, ANDISORD4 is applied to 5×5×5 control volumes with average and local heat fluxes reported in Table 1(*a*) and Fig. 2. Note that the results for these coarse grids may not be grid independent.

Modeling the side walls as diffusely reflecting walls (producing the results shown in Fig. 2 and Table 1*a*) should produce averaged heat fluxes on the receiving wall that are smaller than those for the parallel-plate geometry. The reason is that part of the energy from the hot wall reaching the side walls (which in the one-dimensional geometry would arrive at the cold wall) is redirected back to the hot wall, resulting in a net decrease in the total energy arriving at the receiving wall. The average heat fluxes, therefore, do not provide enough information about the accuracy of the modeling.

When a one-dimensional geometry is modeled by a three-dimensional model with a large aspect ratio (classical approach), the local heat flux at the center should be the closest

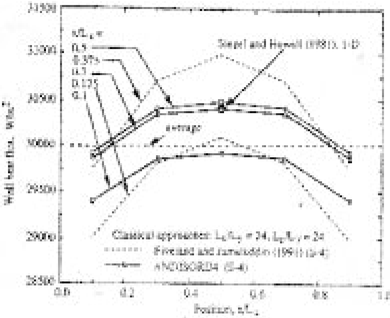

Fig. 2 Local wall heat fluxes to receiving wall

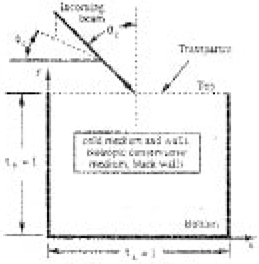

Fig. 3 Geometry for Test 2

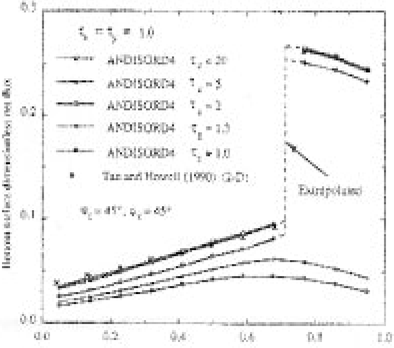

Fig. 4 Classical approach: bottom heat fluxes at $z = L/2$

indication of the heat flux for the one-dimensional geometry. At this central location, end effects are minimized. Table 1(*b*) presents the heat flux at the central location of the receiving surface as predicted by Fiveland and Jamaluddin (1991) and ANDISORD4. Fiveland and Jamaluddin (1991) overestimate the heat flux reported by Siegel and Howell (1981). Good agreement between the result from ANDISORD4 and that of Siegel and Howell (1981) is shown.

Mirror Approach. When the mirror technique is used, ANDISORD4 produces a uniform heat flux of value shown in Table 1(*b*). The heat flux does not change when the aspect ratio is varied from 1:1:1 to 24:1:24. The heat flux indicates that ANDISORD4 overpredicts the heat flux of Siegel and Howell (1981) by less than 1 percent.

Shortcut Approach. For one-dimensional modeling using the shortcut approach, the results presented in Table 1(*b*) are obtained. The error (compared to that of Siegel and Howell, 1981) in the heat flux is less than 1 percent and the heat flux profiles are uniform.

Remarks. From the preceding discussion, it can be concluded that:

1 When the one-dimensional geometry is modeled with the classical approach, the local heat flux at the center of the receiving plate evaluated with ANDISORD4 correctly estimates the averaged heat flux.

2 Most of the energy transfer takes place in the transparent bands. Siegel and Howell (1981) report 97.46 percent of the total heat exchange as taking place in these bands. Computations using ANDISORD4 yielded a value of 97.59 percent. Discrete-ordinate codes are known to have difficulties in situations involving transparent media. These difficulties, known as ray effects, are mainly due to numerical diffusion as a consequence of coarse directional discretization for the problem and become important in two-dimensional and three-dimensional geometries (Gerstl and Zardecki, 1985; Sanchez and Smith, 1992). The applicability of the classical approach in situations involving large optical thickness is thus questionable.

3 The shortcut approach is numerically more efficient than the mirror method. On a personal computer, the shortcut approach needed 125 s while the mirror technique needed 916 s to solve the same problem. This numerical efficiency and accuracy and the fact that no modification of the original code is required make the shortcut approach superior to the classical and mirror approaches.

Test 2. The geometry for Test 2 is shown in Fig. 3. The medium is cold, gray, and contained in a parallelepiped with a transparent top wall; the other enclosure walls are cold, opaque, and black. The length of the domain in the z direction is infinite. Scattering is isotropic and conservative.

Classical Approach. Starting with a cubical enclosure of unit optical thickness ($\tau_x = \tau_y = \tau_z = 1$), the optical thickness in the z direction is increased in ANDISORD4 to approach at $\tau_z/2$ the two-dimensional of Tan and Howell (1990). Results for five different values of τ_z are presented in Fig. 4, where the dimensionless heat fluxes are defined as the net fluxes divided by the heat flux at the top wall due to the parallel beam. For $\tau_z = 1.0$ and 1.5, the entire bottom surface is in a shadow. The two-dimensional results, including shadowing effects, are essentially represented by those for $\tau_z = 5$.

Mirror and Shortcut Approaches. Figure 5 shows results obtained when either the mirror or the shortcut approaches are applied. The results obtained for both approaches show excellent agreement with those from Tan and Howell (1990). The shadowing effects are also represented.

Remarks. The classical approach, applied to 11 × 11 × 11 control volumes and with $\tau_z = 20$, required a wall clock time of 780 s to solve the problem. The mirror and shortcut ap-

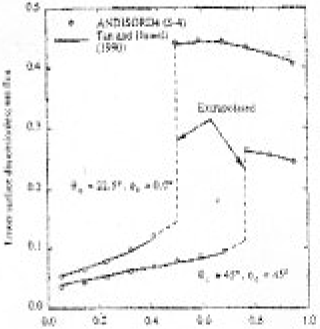

Fig. 5 Mirror or shortcut approach for Test 2

respectively, to achieve the same results.

Conclusions

The capability of a three-dimensional radiative transfer model based on the discrete-ordinates method to accommodate lower order dimensionality by setting to zero, after the evaluation of the scattering phase function, the direction cosine(s) corresponding to the infinite length(s) of the domain is verified. The shortcut approach to the lesser dimensionality issue is tested and compared to the classical and mirror approaches. From the examples presented, the shortcut approach allows for the solution of one-dimensional, two-dimensional, and three-dimensional problems by means of a single computer algorithm. Of particular importance is the fact that in all three dimensionalities, parallel beam and detectors can be included without any alteration of the code.

When compared with the classical and mirror approaches, the shortcut approach is noticeably faster and less memory demanding. In optically thick applications, if a three-dimensional procedure is employed, the shortcut approach is the only viable alternative to the lesser dimensionality problem. End effects, which can be important in the classical approach, are not a concern in either the mirror or the shortcut approaches.

Acknowledgments

This study was partially supported by grant No. NA89AA-D-AC195 from the National Oceanic and Atmospheric Administration: Global Change Program. The first author would also like to acknowledge the financial support received from Universidad de los Andes and Fundación Gran Mariscal de Ayacucho, both from the Republic of Venezuela.

References

Carlson, B. G., and Lathrop, K. D., 1968, "Transport Theory: The Method of Discrete-Ordinates," *Computing Methods in Reactor Physics*, H. Greenspan, C. N. Kelber, and D. Okrent, eds., Gordon and Breach, New York, pp. 171–266.

Fiveland, W. A., 1991, "The Selection of Discrete Ordinate Quadrature Sets for Anisotropic Scattering," *Fundamentals of Radiation Heat Transfer*, W. A. Fiveland, A. L. Crosbie, A. M. Smith, and T. F. Smith, eds., ASME HTD-Vol. 160, pp. 89–96.

Fiveland, W. A., and Jamaluddin, A. S., 1991, "Three-Dimensional Spectral Radiative Heat Transfer Solutions by the Discrete-Ordinates Method," *J. Thermophysics and Heat Transfer*, Vol. 5, No. 3, pp. 335–339.

Gerstl, S. A. W., and Zardecki, A., 1985, "Discrete-Ordinates Finite-Element Method for Atmospheric Radiative Transfer and Remote Sensing," *Applied Optics*, Vol. 24, No. 1, pp. 81–93.

Haferman, J. L., Krajewski, W. F., Smith, T. F., and Sánchez, A., 1993, "Radiative Transfer for a Three-Dimensional Raining Cloud," *Applied Optics*, Vol. 32, No. 15, pp. 2795–2802.

Kim, T. K., and Lee, H. S., 1989, "Radiative Transfer in Two-Dimensional Anisotropic Scattering Media With Collimated Incidence," *J. Quant. Spectrosc. Radiat. Transfer*, Vol. 42, No. 3, pp. 225–238.

Kim, T. K., and Lee, H. S., 1990, "Modified δ-M Scaling Results for Mie-Anisotropic Scattering Media," ASME JOURNAL OF HEAT TRANSFER, Vol. 112, pp. 988–994.

Sánchez, A., Smith, T. F., and Krajewski, W. F., 1991, "Three-Dimensional Radiative Heat Transfer in a Parallelepiped With Collimated Incident Source," *Fundamentals of Radiation Heat Transfer*, W. A. Fiveland, A. L. Crosbie, A. M. Smith, and T. F. Smith, eds., ASME HTD-Vol. 160, pp. 27–36.

Sánchez, A., and Smith, T. F., 1992, "Surface Radiant Exchange for Two-Dimensional Rectangular Enclosures Using the Discrete-Ordinates Method," ASME JOURNAL OF HEAT TRANSFER, Vol. 114, pp. 465–472.

Sánchez, A., Krajewski, W. F., and Smith, T. F., 1992, "A General Purpose Radiative Transfer Model for Application to Remote Sensing in Multi-dimensional Systems," IIHR Report No. 352, Iowa Institute of Hydraulic Research, The University of Iowa, Iowa City, IA.

Siegel, R., and Howell, J. R., 1981, *Thermal Radiation Heat Transfer*, Hemisphere Publishing Corporation, Washington, DC.

Stamnes, K., Tsay, S.-C., Wiscombe, W., and Jayaweera, K., 1988, "Numerically Stable Algorithm for the Discrete-Ordinate-Method Radiative Transfer in Multiple Scattering and Emitting Layered Media," *Applied Optics*, Vol. 27, No. 12, pp. 2502–2509.

Tan, Z., and Howell, J. R., 1990, "Two-Dimensional Radiative Heat Transfer in an Absorbing, Emitting, and Linearly Anisotropic Scattering Medium Exposed to a Collimated Source," *Radiation Heat Transfer: Fundamentals and Applications*, eds., ASME HTD-Vol. 137, pp. 101–110.