RADIATIVE TRANSFER COMPUTATIONS IN CYLINDRICAL COORDINATES: A PARALLEL IMPLEMENTATION OF THE DISCRETE ORDINATES METHOD

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SUMMARY

In this paper, a numerical model for solving the Radiative Transfer Equation in cylindrical coordinates by means of the Discrete Ordinates Method has been parallelized using several platforms and parallelization strategies. A parallelized High Performance Fortran (HPF) version was implemented on an Origin 2000 platform with 4 nodes; a LINDA parallel version was run on an IBM SP2 computer with 8 nodes; and finally a Message Passing Interface (MPI) version based on the quadrature set was implemented on both computers. The results from all versions and platforms were compared and the goodness of the MPI implementation was asserted.

INTRODUCTION

Radiative Transfer is present in a wide variety of engineering applications ranging from furnace design and combustion analysis to architectural comfort studies and satellite remote sensing (Sánchez et al., 1992). In these applications, the Radiative Transfer Equation (RTE), has to be solved. Because of its many advantages, the Discrete Ordinates Method (DOM) (Carlson, 1953) has become the most widely used procedure –almost standard– for the solution of the RTE.

The need to solve bigger and more calculation-intensive problems has resulted in the recent development of parallel algorithms of the DOM (Haghighat, 1991; Hannebutte and Lewis, 1992; Haferman, 1995; Gonçalves, and Coelho, 1997). These parallel algorithms are, normally, modifications of previously existing serial implementations. However, the Standard Solution Procedure (SSP) normally suggested in the literature for serial applications of the DOM to cylindrical coordinate systems, is based on quadrants (Fiveland, 1982; Jendoubi, 1991). This approach becomes a very interdependent procedure that does not allow for any practical parallelization scheme when working with Message Passing Parallelizing Schemes (MPPS) over Distributed Memory Machines (DMM).

In this paper, a numerical model for solving the Radiative Transfer Equation in cylindrical coordinates by means of the Discrete Ordinates Method has been parallelized using several platforms and parallelization strategies. As a first step toward parallelization, the serial code is modified and a New Solution Procedure (NSP) based on the existence of Sets of Independent Directions (SID) is presented. Next, a parallelized High Performance Fortran (HPF) version was implemented on an Origin 2000 platform with 4 nodes; a LINDA parallel version was run on an IBM SP2 computer with 8 nodes; and finally a Message Passing Interface (MPI) version based on the quadrature set was implemented on both computers. The parallelizing strategy for the MPI version is based on the quadrature set –i.e. the number of directions to be solved– and it is designed to minimize the interdependence between directions.

A model problem involving participating media was solved using the different versions of the code. Various combinations of quadrature sets and number of processors were evaluated. In each case, acceleration and efficiency were estimated and the goodness of the MPI strategy was asserted.

MATHEMATICAL MODEL

The radiative transfer equation (RTE), describing the propagation of monochromatic radiation intensity (I) along a line of sight (ζ), can be written as:

$$\frac{dI(\zeta,\omega)}{d\zeta} = -(a+s)I(\zeta) + aI_b(\zeta,T) + \frac{s}{4\pi}\int_{4\pi} I(\zeta,\omega_i)\,\Phi(\omega,\omega_i)\,d\omega_i \qquad (1)$$

where a and s are the monochromatic absorption coefficient and the scattering coefficient, respectively. I_b is the black body emitted intensity; Φ is the phase function; T represents temperature; and ω is the solid angle.

Eq. (1) is subjected (for diffuse surfaces) to the following boundary conditions:

$$I^{+}(\zeta,\omega) = \varepsilon I_b^{+} + \frac{\psi}{\pi}\int_{2\pi} I^{-}(\zeta,\omega_i)\,\alpha\,d\omega_i \qquad (2)$$

In Eq. (2), superscripts +, and - indicate radiation going from the boundary toward the medium inside the domain, and from the medium toward the boundary respectively. α represents the cosine of the angle between the direction of propagation of I_λ and the normal to the given boundary. The reflectance is denoted by ψ, and is understood to be monochromatic.

The problem of interest in this study is that of two-dimensional, cylindrically axisymmetric geometries as depicted in Figure 1.

When applied to a discrete set of directions in this geometry, Eq. (1) and Eq. (2) become (Tsai and Özisik, 1990; Jendoubi, 1991):

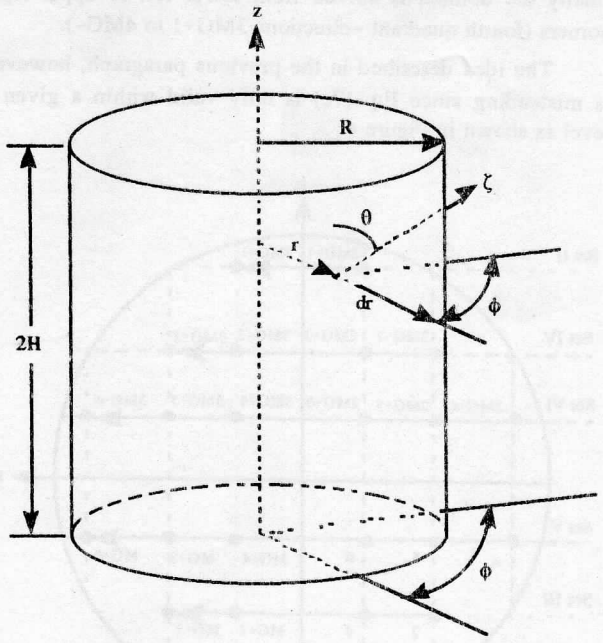

Figure 1 – Geometry

$$\frac{\mu_m}{r}\frac{\partial[r\,Im]}{\partial r} - \frac{1}{r}\frac{\partial[\eta_m\,Im]}{\partial \phi} + \xi_m\frac{\partial Im}{\partial z} = -(a+s)Im + aI_b(T) + \frac{s}{4\pi}\sum_{m^i} Wm^i\,Im^i\,\Phi(m,m^i) \qquad (3)$$

$$Im^+ = \varepsilon\,I_b^+ + \frac{\psi}{\pi}\sum_{m^i} Wm^i\,\mu_m{}^i\,Im^i \quad \text{at } r=R \qquad (4)$$

where $0 \le r \le R$, R being the cylinder radius; $Im = I(r,\omega)$; $Im^i = I(r,\omega_i)$; m and m^i represent discrete directions; μ_m, η_m, and ξ_m are the direction cosines for direction m; Wm^i and $\mu_m{}^i$ are the weight and direction cosine for direction m^i, respectively; and $\Phi(m,m^i)$ is the discretized scattering phase function between directions m and m^i. The boundary at $r = 0$ is treated as a specular boundary.

THE DISCRETE ORDINATES METHOD (S-N)

The Discrete Ordinates Method (DOM or S-N) consists in the selection of an appropriate set of directions on which to solve Eq. (3) and Eq. (4). The procedures for selecting the quadrature of integration (directions), obtaining the direction cosines and associated weights, or for evaluating the discretized phase function are well described elsewhere (Carlson, 1953; Tsai and Özisik, 1990; Fiveland, 1991; Jendoubi, 1991; Sánchez et al., 1992). For the symmetric problem being treated here, the order of the quadrature is defined by the value of N according to the following expression:

$$MG = \frac{N(N+2)}{8} \qquad (5)$$

where MG is the number of discrete directions to be solved per quadrant. Furthermore, all the directions are distributed, in the whole hemisphere, in N levels. Consequently, S-2 requires the solution of one direction per quadrant distributed in two levels in the whole hemisphere, S-4 requires 3 directions per quadrant and 4 levels in the hemisphere, S-6 requires 6 directions per quadrant in six levels, and so on. The distribution of quadrants, and the directions contained in each quadrant are shown in Figure 2.

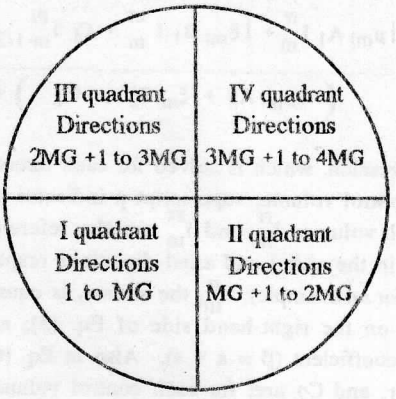

Figure 2 – Distribution of quadrants and directions contained in each quadrant

In order to find the discrete solution of Eq. (3), the whole domain is subdivided into a series of MRxMZ control volumes –not necessarily equal– as depicted in Figure 3a.

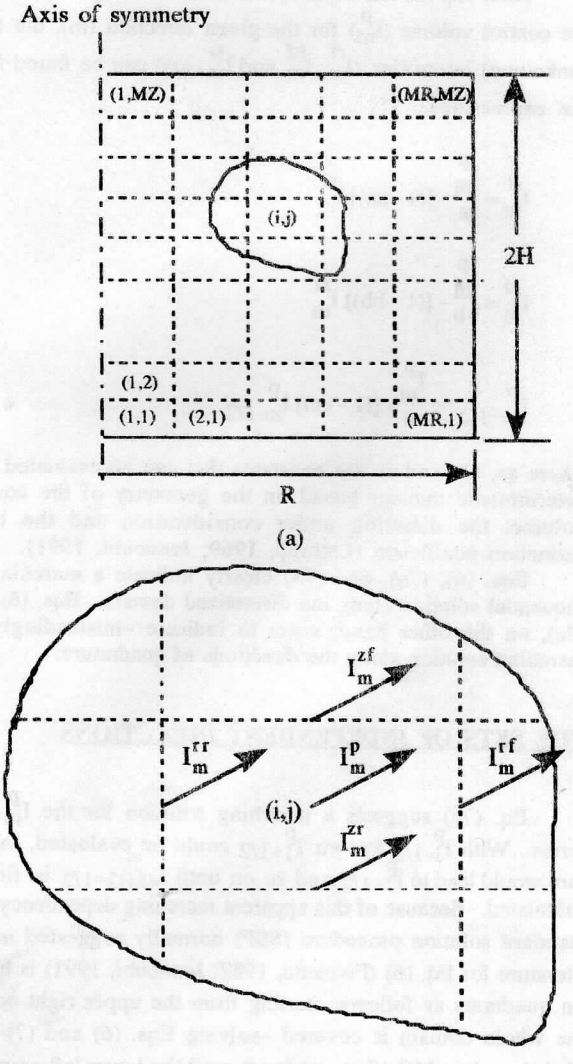

Figure 3 – (a) Discretized domain. (b) Example solution for a direction in the fourth quadrant

After multiplying Eq. (3) by dv = 2πrdrdz, and integrating over a given control volume the following expression is obtained:

$$I_m^p = \frac{|\mu_m| A_1 I_m^{rr} + |\xi_m| B_1 I_m^{zr} + C_1 I_{m-1/2}^p + \beta S_m^p}{\left(|\mu_m| A_2 + |\xi_m| B_2 + C_2 \right) + \beta} \quad (6)$$

in this expression, which is solved for each direction (m) and for each control volume, superscript p indicates average over the control volume; I_m^{rr} and I_m^{zr} are the reference (known) intensities in the radial and axial directions respectively (see Figure 3b for an example); S_m^p, the source, is equal to the last two terms on the right-hand side of Eq. (3); and β is the extinction coefficient (β = a + s). Also in Eq. (6) : A_1, A_2, B_1, B_2, C_1, and C_2 are, for each control volume, constants that depend on the geometry, the direction being considered, and the local extinction coefficient. These constants can be evaluated in a deterministic manner for each particular problem (Lathrop, 1969; Jendoubi, 1991) and are not relevant to this study. Of particular importance to this work is the term $I_{m-1/2}^p$ to be explained later.

After Eq. (6) has been solved for the average intensity in the control volume (I_m^p) for the given direction (m), the final (unknown) intensities (I_m^{rf}, I_m^{zf}, and $I_{m+1/2}^p$) can be found from the expressions:

$$I_m^{rf} = \frac{I_m^p}{aa} - [(1 - aa)] I_m^{rr} \quad (7a)$$

$$I_m^{zf} = \frac{I_m^p}{bb} - [(1 - bb)] I_m^{zr} \quad (7b)$$

$$I_{m+1/2}^p = \frac{I_m^p}{cc} - [(1 - cc)] I_{m-1/2}^p \quad (7c)$$

where aa, bb, and cc are constants that can be evaluated in a deterministic manner based on the geometry of the control volume, the direction under consideration and the local extinction coefficient (Lathrop, 1969; Jendoubi, 1991).

Eqs. (6), (7a), and (7b) clearly indicate a marching or sequential solution along the discretized domain. Eqs. (6) and (7c), on the other hand, seem to indicate –misleadingly– a marching solution along the directions of quadrature.

THE SETS OF INDEPENDENT DIRECTIONS

Eq. (7c) suggests a marching solution for the $I_{m+1/2}^p$ terms. With $I_{1-1/2}^p$ known $I_{1+1/2}^p$ could be evaluated, this in turn would lead to $I_{2+1/2}^p$ and so on until $I_{4MG+1/2}^p$ is finally calculated. Because of this apparent marching dependency, the standard solution procedure (SSP) normally suggested in the literature for Eq. (6) (Fiveland, 1982; Jendoubi, 1991) is based on quadrants as follows: starting from the upper right corner the whole domain is covered –solving Eqs. (6) and (7)– for directions 1 to MG (first quadrant) until the lower left corner is reached, next the domain is solved from the lower right corner to the upper left for directions 2MG+1 to 3MG (third quadrant), then the procedure continues from the upper left corner to the lower right (second quadrant –directions MG+1 to 2MG–), and finally the domain is solved from lower left to upper right corners (fourth quadrant –directions 3MG+1 to 4MG–).

The idea described in the previous paragraph, however, is misleading since Eq. (7c) is only valid within a given ξ level as shown in Figure 4.

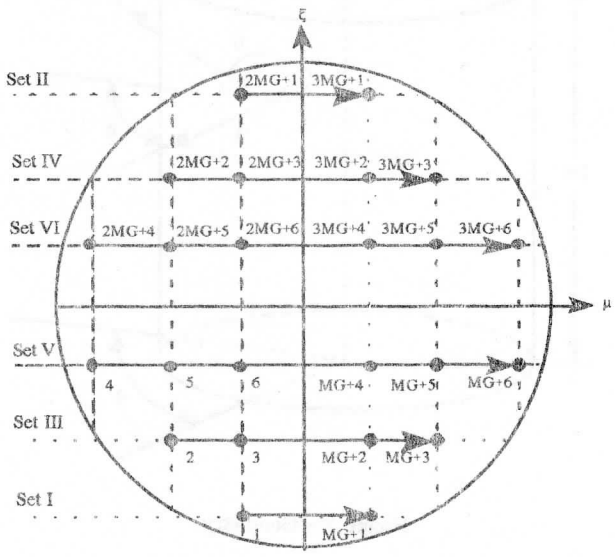

Figure 4 – Sets of independent directions for an S-6 implementation of the DOM

In Figure 4, each arrow indicates a level and the direction for the marching procedure of Eq. (7c). In general, a given S-N implementation of the DOM has N ξ levels or SIDs. The smallest set includes two directions (sets I and II) and the largest set includes N directions as shown in Table 1.

It should be noticed that all sets contain mirroring directions in two different quadrants. For example, Set III contains mirroring directions in the first and second quadrant while Set IV includes mirroring directions in the third and fourth quadrant. In general, odd numbered sets relate first and second quadrants, while even numbered sets relate the third and the fourth quadrant.

Based on the previous description of SIDs, a new solution procedure (NSP) for Eqs. (6) and (7) can be proposed as follows:

1 The domain is divided into MRxMZ control volumes and the order of quadrature (N) is selected.
2 The solution –Eqs. (6) and (7)– proceeds by sets, starting with Set I and ending with Set N.
3 For odd numbered sets, the domain is swept first from the upper right corner to the lower left corner (first quadrant) and then from the upper left corner to the lower right corner (second quadrant).
4 For even numbered sets, the domain is swept first from the lower right corner to the upper left corner (third quadrant) and then from the lower left corner to the upper right corner (fourth quadrant).
5 In all cases, and for the first direction (m) of each set,

$$I_{m-1/2}^p = I_m^p$$

This NSP allows the application of Eq. (7c) in a straight forward manner within each level and avoids the need for storing intermediate values of $I_{m-1/2}^p$. Further, since each set contains mirroring directions between two adjacent quadrants, the symmetric left boundary of the domain can be updated instantaneously while solving for each individual set.

It should be pointed out that if the resulting algorithm is to be parallelized based on Message Passing Parallelizing Schemes (MPPS) over Distributed Memory Machines (DMM) then,

because of its minimum requirement for message passing among sets, clearly, the NSP lends itself to a parallelization scheme based on sets. Moreover, knowing the number of directions to be solved in each set, the load distribution among processors can be easily optimized.

Table 1 – Sets of Independent Directions

N	Sets	Directions
≥ 2	I	1 ⇒ MG+1
	II	2MG+1 ⇒ 3MG+1
≥ 4	III	2,3 ⇒ MG+2,MG+3
	IV	2MG+2,2MG+3 ⇒ 3MG+2,3MG+3
≥ 6	V	4,5,6 ⇒ MG+4,MG+5,MG+6
	VI	2MG+4,2MG+5,2MG+6 ⇒ 3MG+4,3MG+5,3MG+6
⋮	⋮	⋮ ⋮ ⋮
=K	K-1	MG-K/2+1,MG-K/2+2 ····,MG ⇒ 2MG-K/2+1,2MG-K/2+2····,2MG
	K	3MG-K/2+1,3MG-K/2+2 ····,3MG ⇒ 4MG-K/2+1,4MG-K/2+2 ····,4MG

PARALLELIZING STRATEGIES AND PLATFORMS

An algorithm for solving the Discrete Ordinates Method in cylindrical coordinates based on the NSP was parallelized according to quadrature, and implemented on two different platforms: an IBM-SP2 with 8 processors and an Origin 2000 with four processors. In each platform, a self parallelizing – automatic– library was used first: the library LINDA was used for the SP2 and the POWER FORTRAN ACCELERATOR library was used for the Origin 2000. Next, an algorithm based on the Message Passing Interface (MPI) library was developed and implemented on both platforms. The MPI implementation was written to work in one of the two following approaches:

1) Master-slaves. In this approach, one of the processors - the master- will not perform any work other than distribution of data, processing of partial results from the other processors, checking convergence, and printout of final results. The other processors -the slaves- will do all the computation of intensities and boundary conditions needed and send, at the end of each iteration, their partial results (the source) to the master. The work load is distributed as evenly as possible (according to Table 1) among the slave processors.
2) All working. In this case, all processors do, basically, the same amount of computing work. The work load is distributed as evenly as possible among all processors.

Therefore, we have a total of two platforms with three parallel implementations on each one.

TEST PROBLEM AND RESULTS

In order to check the goodness of the different strategies and platforms described in the previous section a test problem was selected. The problem is that of a finite black cylinder of radius R = 1 m and height 2H = 10 m. The side walls are hot (emissive power equal to a positive constant) while the end walls are cold, and the medium inside the cylinder is gray, cold, scatters radiation isotropically, and has extinction and absorption coefficients of 1.0 and 0.3 respectively. The problem is considered converged when the addition of the changes –in percentage- of all the intensities between two iterations, is less than or equal to 0.0001.

Figure 5 shows the normalized mid-plane irradiation obtained with the algorithm developed in this work. The results are in good agreement with those reported by Jendoubi (1991) for the same problem and also agree (given the large aspect ratio) with the results reported for the equivalent one-dimensional problem (Siewert and Thomas, 1985;Tsai and Özisik, 1990). It should be mentioned here, that the solution of this problem required 21 iterations when applying the SSP and only 16 iterations when the NSP was used.

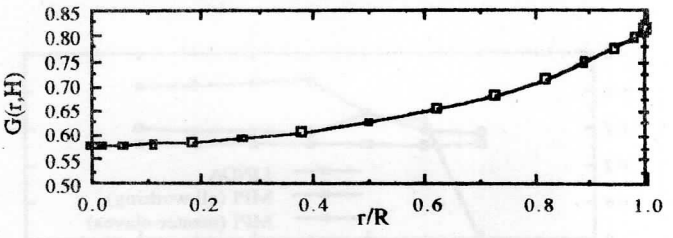

Figure 5 – Normalized mid-plane irradiation using S-6 and 15x15 control volumes.

In order to assert the goodness of the different parallelizing strategies and platforms, the concepts of acceleration (Acc.) and efficiency (η) commonly used by parallel computing specialists were redefined in a way more suitable to the engineering community. These definitions are as follows:

$$\text{Acc.} = \frac{\text{Time (1 Processor)} - \text{Time (N Processors)}}{\text{Time (1 Processor)}} \quad (8)$$

$$\eta = \frac{\text{Acceleration achieved with N Processors}}{\text{Maximun acceleration possible with N Processors}} \quad (9)$$

where the denominator in Eq. (9) is:

$$\text{Maximum Acc. possible with N Processors} = 1 - \frac{1}{N} \quad (10)$$

Figures 6 and 7 show the acceleration achieved for the different strategies on the SP2 and Origin 2000 respectively. The results shown are for the maximum granularity possible in our systems (S-14 and 50x50 control volumes) due to memory limitations. For finer granularity the Acceleration was insignificant and even negative (S-6 and 15x15, for example).

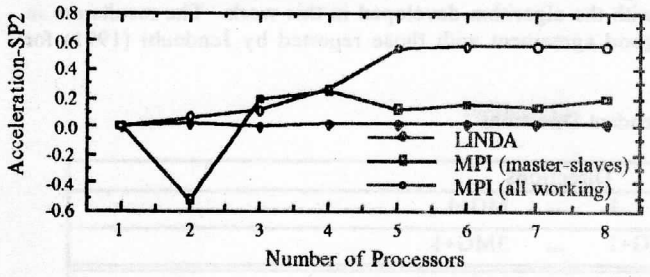

Figure 6 – Acceleration on the SP2. (S-14 and 50x50 control volumes).

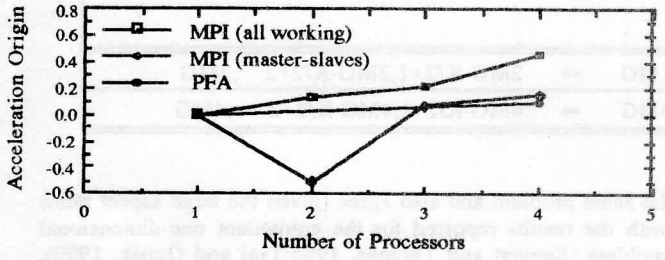

Figure 7 – Acceleration on the Origin 2000. (S-14 and 50x50 control volumes).

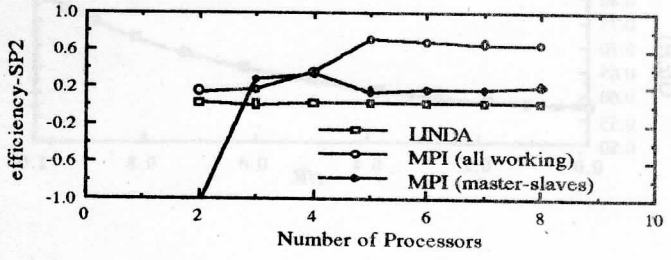

Figure 8 – Efficiency on the SP2. (S-14 and 50x50 control volumes).

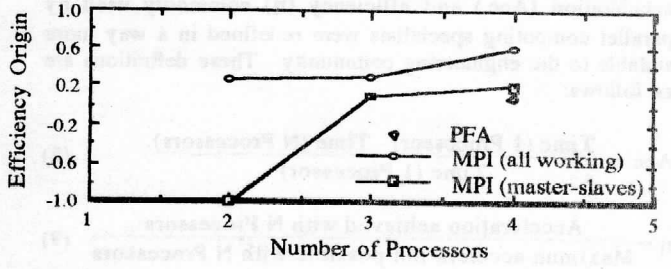

Figure 9 – Efficiency on the Origin 2000. (S-14 and 50x50 control volumes).

Figures 6 to 9 show that automatic parallelization (LINDA on the SP2 and HPF on the Origin) failed to produce good results. The poor efficiencies and accelerations obtained are a clear indication of the strong dependency implicit in the code which automatic libraries are not able to solve.

Negative accelerations and efficiencies appeared, when working with two processors, for the MPI implementation based on the master-slaves strategy. This is due to the fact that all computations are being handled by one processor (just like in the serial case) while the presence of the other processor only implies the need for communications. Therefore, this deceleration is a direct consequence of the communications needed in the parallel version.

The maximum accelerations obtained for the master-slave strategy were 25.7 % for the SP2 with 4 processors and 15 % for the Origin 2000, also with 4 processors. The maximum efficiencies were found for the same working conditions and corresponded to 34.3 % and 20 % respectively.

For the "all-working" strategy on the other hand, maximum acceleration (56 %) and efficiency (69 %) corresponded, on the SP2, to the cases with 5 and 6 (or more) processors respectively. The same strategy on the Origin 2000 reported a maximum acceleration of 40 % for 4 processors and a maximum efficiency of 60 % for the same number of processors.

CONCLUSIONS

In this paper, a new solution procedure for the Discrete Ordinates Method in cylindrical coordinates has been presented. The procedure is based on sets of independent directions and is particularly important for parallelization strategies based on message passing libraries. In particular, minimizing message passing between processors and optimizing work-load-distribution among the same processors are, now, possible tasks when parallelizing based on quadrature sets..

The serial code has been parallelized on two platforms and with several strategies.

The tests performed show that automatic parallelization of the code produces very poor results due to the inter-dependency of the procedure. Thus, Message Passing Interfaces seem to be the recommended path for this type of problems.

Two different strategies were tested with MPI: master-slaves and "all-working". The former is easier to implement but it gives poorer results. In the experiments performed in this work, the "all-working" strategy produced efficiencies close to 70 % and accelerations superior to 50 %.

Finally, it should be mentioned again that the results obtained were strongly affected by the granularity (amount of work done by each processor) of the problem. As the granularity increases the parallel implementation becomes more efficient.

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