

A GENERAL PURPOSE RADIATIVE TRANSFER MODEL FOR APPLICATION TO  
REMOTE SENSING IN MULTI-DIMENSIONAL SYSTEMS

by

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A thesis submitted in partial fulfillment  
of the requirements for the Doctor of  
Philosophy degree in Mechanical Engineering  
in the Graduate College of  
The University of Iowa

December 1991

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## ABSTRACT

The overall objective of this study is the development, implementation, and validation of a radiation transfer model with applicability to one-, two-, and three-dimensional rectangular domains. The formulation and implementation of the discrete ordinates method, selected as the method to be used in the development of the model, is discussed in detail. The discussion includes: quadrature selection; scattering phase function modeling; spatial and directional discretization of the governing equations; modeling of lesser dimensionality domains by means of a three-dimensional code; implementation of a collimated source; and evaluation of output intensities in any desired direction, among others. The resulting algorithm is capable of accounting for: black, gray, specularly reflecting, and spectrally emitting boundaries with any given temperature distribution; symmetric and/or periodic boundaries; spectral and spatial variations of radiatively participating gases and polydispersions (absorbing, emitting, and anisotropically scattering media) under optically thin or optically thick conditions; non-homogeneous media; radiative equilibrium on a gray or spectral basis; internal energy sources; isotropic background radiation source; incoming collimated source from any direction and acting on any set of boundaries; output intensity toward an instrument placed in any desired direction; and easy coupling with other models performing fluid flow calculations. The final product of this work is a set of two computer programs capable of solving for all of the above, and complemented with the ability to perform radiative properties calculations for polydispersions with renormalized phase functions according to any desired order of quadrature. The computer package is specially designed for easy use, computational

efficiency; accuracy, stability, and the potential to take advantage of modern parallel and vector machines. The model is particularly suited, but not limited, to remote sensing applications.

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